Note on the shape circularity measure method based on radial moments

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Abstract

In this note we show that the, so called, circularity measures based on radial moments, as defined in [1], are a particular case of the circularity measures introduced by [2].

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1 Introduction

A family of circularity measures $\mathcal{C}_\beta(S)$ was introduced recently in [2]. More precisely, if S denotes a planar shape, $\mu_{0,0}(S)$ is the area of S, and β is a number from the interval $(-1,\infty)$, then the quantities $\mathcal{C}_\beta(S)$ indicate/measure how much the considered shape S differs from a planar circular disc, of the same area as the given shape S. The formal definition, of the circularity measures $\mathcal{C}_\beta(S)$, is as follows.

Definition 1 *Let S be a given shape whose centroid coincides with the origin and a real β such that $-1 < \beta$ and $\beta \neq 0$. Then the circularity measure $\mathcal{C}_\beta(S)$ is defined as*

$$\mathcal{C}_\beta(S) = \begin{cases} \dfrac{\mu_{0,0}(S)^{\beta+1}}{(\beta+1)\pi^\beta \iint_S (x^2+y^2)^\beta dxdy}, & \beta > 0 \\[2ex] \dfrac{(\beta+1)\pi^\beta \iint_S (x^2+y^2)^\beta dxdy}{\mu_{0,0}(S)^{\beta+1}}, & \beta \in (-1,0). \end{cases} \tag{1}$$

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The formula in (1) is given in Cartesian coordinates. If the polar coordinate system is involved (instead of the Cartesian coordinate system):

$x = r \cdot cos\theta, \quad y = r \cdot \sin\theta$ (the Jacobian for this coordinate transformation is $|J| = r$)

then

$$\iint_S (x^2+y^2)^\beta \, dx \, dy \; = \; \int_\theta \int_r ((r\cdot cos\theta)^2 + (r\cdot\sin\theta)^2)^\beta \cdot r \, dr \, d\theta \; = \; \int_\theta \int_r r^{2\beta+1} \, dr \, d\theta$$

and consequently (1) becomes

$$\mathcal{C}_\beta(S) = \begin{cases} \dfrac{\mu_{0,0}(S)^{\beta+1}}{(\beta+1)\pi^\beta \int_\theta \int_r r^{2\beta+1} \, dr \, d\theta}, & \beta > 0 \\[2ex] \dfrac{(\beta+1)\pi^\beta \int_\theta \int_r r^{2\beta+1} \, dr \, d\theta}{\mu_{0,0}(S)^{\beta+1}}, & \beta \in (-1,0). \end{cases} \tag{2}$$

Now, by setting $\beta = \frac{p}{2}$, the first expression (for $\beta > 0$) in (2) becomes

$$\mathcal{C}_\beta(S) = \mathcal{C}_{p/2}(S) = \frac{\frac{2}{p+2} \cdot \pi^{-p/2} \cdot \mu_{0,0}(S)^{\frac{p+2}{2}}}{\int_\theta \int_r r^{p+1} \, dr \, d\theta}. \tag{3}$$

Circularity measures based on radial moments, introduced in [1], are denoted by $\zeta_p(D)$ and formally defined, by the expression in (9) from [1], as

$$\zeta_p(D) \; = \; \frac{\frac{2}{p+2} \cdot \pi^{-p/2} \cdot [u_0(D)]^{\frac{p+2}{2}}}{u_p(D)}. \tag{4}$$

Further, [1] uses the following denotation

- $u_p(D) \; = \; \iint_D (r - \bar{r})^p ds,$ with $ds = r \cdot dr \cdot d\theta \quad \bar{r} = \sqrt{x_c^2 + y_c^2},$ and

  $(x_c, y_c) \; = \; \left(\dfrac{\iint_D x \; ds}{\iint_D \; ds}, \dfrac{\iint_D y \; ds}{\iint_D \; ds} \right)$ being the centroid of the considered shape D.

  (Notice: $u_0(D) = \mu_{0,0}(D)$ and $u_p(D) = \int_\theta \int_r r^{p+1} dr d\theta$, if $\bar{r} = 0$, i.e. $(x_c, y_c) = (0,0)$.)

Finally, since $\zeta_p(D)$ is translation invariant (see Theorem 2 from [1]) we can set $\bar{r} = 0$ (i.e we can assume that the shape D is translated such that its gravity center (x_c, y_c) coincides with the origin $(0,0)$), and deduce that (for $p > 0$)

$$\zeta_p(D) \; = \; \mathcal{C}_{p/2}(D). \tag{5}$$

In other words, the formula in (4) is equivalent to the formula in (3), and further, shape circularity measures $\zeta_p(D)$ based on radial moments, from [1], are particular subcases of the family of circularity measures $\mathcal{C}_\beta(S)$, introduced by [2] (measures from [2] are defined for $\beta = \frac{p}{2}$ negative, as well).

It is worth mentioning that the identity in (5) is evident in the experimental results from Table 1 in [1], which includes $\mathcal{C}_p(D)$ denoted by $H_p(D)$. Although there is a systematic offset between $\zeta_p(D)$ and $\mathcal{C}_{p/2}(D)$, possibly caused by digitization and numerical errors, the results for $\zeta_{p=2}(D)$ are similar to $\mathcal{C}_{p=1}(D)$, such that their ratios are all the same to within 3 significant places. Likewise, the ratios of $\zeta_{p=4}(D)$ and $\mathcal{C}_{p=2}(D)$ are the same to within 3 significant places.

References

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