Using a Neural Network to Calculate the Sensitivity Vectors in Synchronisation of Chaotic Maps

by

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Keywords: Synchronisation, Chaos, Control, Otani-Jones method.

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Abstract When parametric control methods such as the OGY [1] or Otani-Jones method [2] are applied to synchronisation, computing the sensitivity vectors at each step poses special problems. In this paper we show that it is possible to use an artificial neural network to calculate the sensitivity vectors when synchronising two chaotic maps.

1. INTRODUCTION

Chaos is characterised by a sensitive dependence of a system's dynamical variables on the initial conditions. Trajectories starting with slightly different initial conditions locally diverge from each other at an exponential rate.

Despite this, it has been established that synchronisation of two chaotic systems is possible [3], and it is an area of increasing interest for its interesting applications. Not only has chaotic synchronisation been successfully applied to secure communications (for example [4]-[7]), but also it is believed that synchronisation plays a crucial role in information processing in living organisms, namely in the rabbit olfactory system, cat visual system and even the monkey neocortex [8]-[11]

If we consider an iterative map F: $\xi_i \rightarrow \xi_{i+1}$, when we use parametric control techniques such as the OGY or Otani-Jones method the state of the system at each step i is evaluated, so that the control parameters p_j can be changed according to the corresponding law. This means that in the case of synchronisation, the sensitivity vectors $s_j = \partial F/\partial p_j$ will have to be calculated at each step, and if the equations of the system are unknown this poses special problems.

The published examples of chaotic synchronisation using these methods [12]-[13] use the Henon map for which the sensitivity vectors are constant.

In this paper we use an artificial neural network to calculate the sensitivity vectors. We implemented the Otani-Jones method to synchronise two Ikeda maps and the parameter used for control has associated with it a sensitivity vector that depends on the state of the system.

The paper is organized as follows. In the first section the Otani-Jones control law is described. Then we describe the training of the neural network and the results of the synchronisation of the Ikeda maps using this neural network. Finally some conclusions and future work is discussed.

2. OTANI-JONES METHOD

One of the principal disadvantages of the OGY control law, in cases where the dynamical equations are not known, is the requirement to estimate the Jacobian J where we have an iterated map F: $\xi_i \rightarrow \xi_{i+1}$. Such estimates can be both time consuming and inaccurate especially when applied to synchronising two chaotic systems.

The Otani-Jones control law [2] attempts to overcome some of these shortcomings. In many situations it is possible to create an effective short term (fast) predicting function $\xi_{i+1} = P(\xi_i)$ for the system which is accurate over the whole (or a large part) of the phase space. In situations where such a short term predictor function P is available the Otani-Jones method can be employed to effect control and it does not require the computation of either f_u (unstable contravariant vector) or λ_u (unstable eigenvalue), although it is still necessary to estimate the sensitivity vectors.

The method proceeds as follows. We assume an accurate short term predictor function $\xi_{i+1} = P(\xi_i)$ is available. Suppose that control parameters $\mathbf{p} = (p_1, ..., p_m)$ are available with nominal value $\mathbf{p}_0$ and that it is required to control the system about a fixed point ξ_F. We can describe the situation by the equation

$$\delta\xi_{i+1}(p) = P\big(\xi_i(p_0)\big) - \xi_F + \delta p_1 \boldsymbol{s}_1 + ... + \delta p_l \boldsymbol{s}_l \tag{1}$$

where $\mathbf{s}_1, ..., \mathbf{s}_m$ are the sensitivity vectors with respect to each control parameter.

Once $\mathbf{s}_1, ..., \mathbf{s}_m$ are known, for any point ξ_j near ξ_F the control law then chooses $\mathbf{p} = (p_1, ..., p_m)$ so as to minimise the squared Euclidean distance

$$|\xi_{i+1}(p) - \xi_F(p_0)|^2 \tag{2}$$

i.e. we choose $\mathbf{p}$ so as to minimise

$$|P\big(\xi_i(p_0)\big) - \xi_F(p_0) + \delta p_1 s_1 + ... + \delta p_l s_l|^2 \tag{3}$$

Let S be the matrix whose column vectors are $\mathbf{s}_1, ..., \mathbf{s}_m$. These vectors are linearly independent (there would seem to be no advantage in choosing a linearly dependent set of sensitivity vectors). The solution to this minimisation problem is given by

$$\delta p = -S^{-1}\Big(P\big(\xi_i(p_0)\big) - \xi_F(p_0)\Big) \tag{4}$$

where S^{-1} is the inverse matrix of S if $l = d$ and the pseudo-inverse otherwise.

In the case of synchronising $\xi_{i+1} = F(\xi_i, \mathbf{p})$ with $\theta_{i+1} = F(\theta_i, \mathbf{p}_0)$ then (1) becomes

$$\delta\xi_{i+1}(p) \equiv \xi_{i+1}(p) - \theta_{i+1}(p_0) \approx P\big(\xi_i(p_0)\big) - P\big(\theta_i(p_0)\big) + \delta p_1 \boldsymbol{s}_1 + ... + \delta p_l \boldsymbol{s}_l \tag{5}$$

and the control law becomes

$$\delta p = -S^{-1}\Big(P\big(\xi_i(p_0)\big) - P\big(\theta_i(p_0)\big)\Big) \tag{6}$$

3. SYNCHRONISATION OF TWO IKEDA MAPS

We have implemented the synchronisation using the Ikeda map which is define by the following equations:

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = F\begin{pmatrix} x_n \\ y_n \end{pmatrix} = \begin{pmatrix} \gamma + R(x_n\cos(T_n) - y_n\sin(T_n)) \\ R(x_n\sin(T_n) - y_n\cos(T_n)) \end{pmatrix} \tag{7}$$

where T_n is

$$T_n = \kappa - \frac{\alpha}{1 + x_n^2 + y_n^2} \tag{8}$$

Taking $\gamma = 0.85$, $\kappa = 0.4$, $\alpha = 5.5$ and $R = 0.9$ the map described by equation (7) (which describes the dynamics of a nonlinear optical cavity [15] produces an attractor similar to the one in .

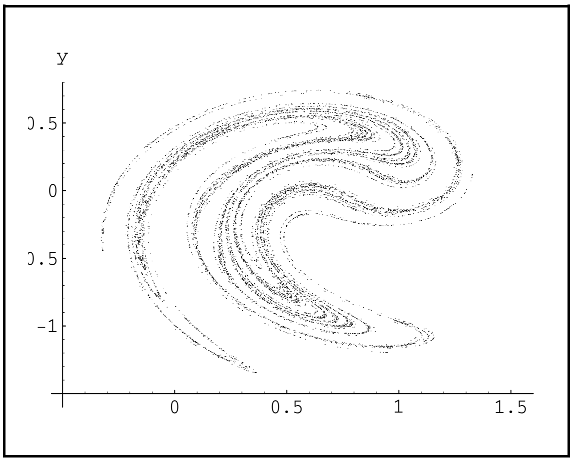

Figure 1 Attractor for the chaotic Ikeda map.

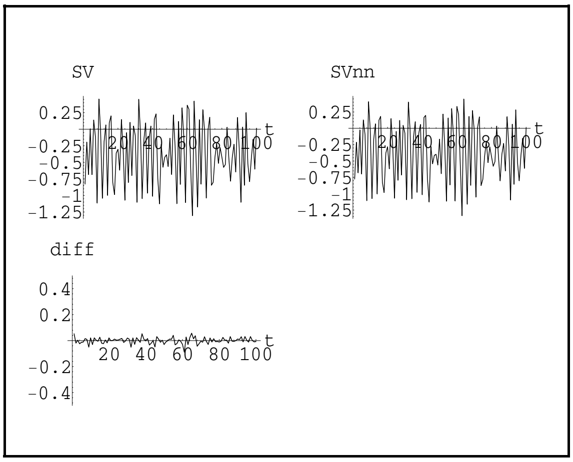

Figure 2 Graphs of the first coordinate of the sensitivity vectors (with the other coordinate the results are similar). Top left: real sensitivity vectors. Top right: NN approximation. Bottom left: the difference between the two.

We will use the parameter *R* for control. The sensitivity vector $\partial F/\partial R$ is then defined by the equation

$$\boldsymbol{s} = \left(x_n\cos(T_n) - y_n\sin(T_n),\ x_n\sin(T_n) - y_n\cos(T_n)\right) \tag{9}$$

and as we can see the vector varies at each point of the trajectory.

A. NN-Calculated Sensitivity Vectors

With the use of the map (7) a set of data was gathered to train a neural network. The neural network used has one input and output layer with 2 neurons each, and 2 hidden layers with 10 neurons each (2-10-10-2) and it was trained with 10000 (actually excellent results can be obtained with far fewer points) training pairs (re-scaled into the range [0.11, 0.89]).

The software we used is basically the conjugate gradient feed-forward network training software from [16] with the modifications for using double precision training to enhance the accuracy and for running under Windows 95 (32-bit). The MSE training error is 0.000024 which easily is sufficient for the synchronisation as we will see in the next section.

In we can see the differences between the real sensitivity vectors and the results of the neural network for testing data.

B. Results

In situations where *F* is unknown we could imagine constructing a fast predictive function $X_{n+1} = P(X_n)$ by training a neural network (see for example [17] in which a network is trained on exactly this map). However, in the present case, where the equations are known and the objective is to demonstrate the technique with respect to variable sensitivity vectors, there is no virtue in training such a network and we shall therefore calculate *P* using *F*.

The Otani-Jones method was used to synchronise two Ikeda maps, using the neural network as an approximation of the sensitivity vectors. The systems

$$X_{n+1} = F(X_n)$$
$$Y_{n+1} = F(Y_n) \qquad (10)$$

started with the following initial conditions: (0.5, -0.8) and (0.001, 0.001), and in and we can see the difference between both variables of the systems become zero.

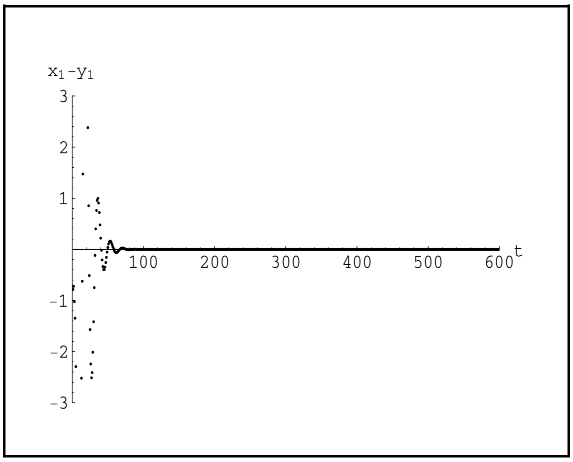

Figure 3 The difference of the time series of the first variable of the two systems.

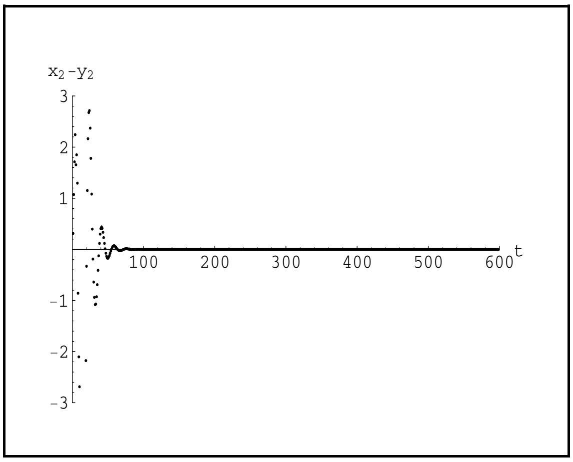

Figure 4 The difference of the time series of the second variable of the two systems.

4. CONCLUSIONS

We propose a simple but effective technique that makes use of an artificial neural network to minimize the calculation effort during synchronisation of maps using parametric control methods.

In this experiment we use (9) to generate the data to train the neural network. If the equations were unknown we could imagine computing sensitivity vectors for several hundred points to generate the training data. This would of course be time-consuming but it would have be done only once.

As future work we will implement this technique in higher dimensional systems and also we will use other parametric control methods such as the OGY.

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