A Definition for Orientation for Multiple Component Shapes

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Abstract. In this paper we introduce a new method for computing the orientation for compound shapes. If the method is applied to single component shapes the computed orientation is consistent with the shape orientation defined by the axis of the least second moment of inertia. If the new method is applied to compound shapes this is not the case, and consequently the presented method is both new and different.

Keywords: Shape, orientation, image normalization, early vision.

1 Introduction

Determining the orientation of a shape is often performed in computer vision so as to enable subsequent analysis to be carried out in the shape's local frame of reference (thereby simplifying that analysis). While for many shapes their orientations are obvious and can be computed easily, the orientation of other shapes may be ambiguous, subtle, or ill defined. The difficulty of the task can be seen from the multiplicity of mechanisms used in human perception in which orientation can be determined by axes of symmetry and elongation [1], as well as cues from local contour, texture, and context [9].

The most common computational method for determining a shape's orientation is based on the axis of the least second moment of inertia [4,6]. Although straightforward and efficient to compute it breaks down in some circumstances – for example, problems arise when working with symmetric shapes [11,12]. This has encouraged the development of other competing methods [2,3,4,5,6,7,10,11]. Suitability of those methods strongly depends on the particular situation in which they are applied, as they each have their relative strengths and weaknesses (e.g. relating to robustness to noise, classes of shape that can be oriented, number of parameters, computational efficiency).

In this paper we focus on the orientation of shape consisting of several components. We introduce a new approach to the shape orientation problem and, after that, we extend the method to shapes that consists of several components. We consider a line that maximises the integral of the squared lengths of the

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projections of line segments whose end points belong to the shape onto this line. Then we define the orientation of the shape by the slope of such a line. It turns out that, if applied to single component shapes, such a method for computing shape orientation is consistent with the standard method based on the line of the least second moment of inertia. Such a new approach leads to a natural definition of the orientation of compound shapes. The computed orientation of compound shapes differs from the orientation of compound shapes by the standard method. In some situations the new compounded shapes is more appropriate than the orientation computed by the standard method.

The paper is organized as follows. In the next section we give a short sketch of the standard method for computing of shape orientation. In Section 3 we introduce a new approach for orienting shapes. Section 4 adopts the new introduced approach to the orientation of compound shapes. Section 5 discusses the properties of the new method and gives some illustrative examples. Section 6 gives concluding comments.

2 The Standard Method

The most standard method for computation of shape orientation is based on the axis of the least second moment of inertia. The axis of the least second moment of inertia ([4,6]) is the line which minimises the integral of the squares of distances of the points (belonging to the shape) to the line. The integral is

$$I(\alpha, S, \rho) = \iint_S r^2(x, y, \alpha, \rho)\ dx\ dy \tag{1}$$

where $r(x, y, \alpha, \rho)$ is the perpendicular distance from the point (x, y) to the line given in the form $X \cdot \sin\alpha - Y \cdot \cos\alpha = \rho$. It is well known that the axis of the least moment of inertia passes through the shape centroid. The centroid is computed as $\left(\frac{m_{1,0}(S)}{m_{0,0}(S)}, \frac{m_{0,1}(S)}{m_{0,0}(S)}\right)$ where $m_{0,0}(S)$ is the zeroth order moment of S and $m_{1,0}(S)$ and $m_{0,1}(S)$ are the first order moments of S. In general, the moment $m_{p,q}(S)$ is defined as $m_{p,q} = \iint_S x^p y^q\ dx\ dy$ and has order $p+q$. Thus, if the shape S is translated by the vector $-\left(\frac{m_{1,0}(S)}{m_{0,0}(S)}, \frac{m_{0,1}(S)}{m_{0,0}(S)}\right)$ (such that the centroid of S coincides with the origin) then it is possible to set $\rho = 0$. Since the squared distance of a point (x, y) to the line $X \cdot \sin\alpha - Y \cdot \cos\alpha = 0$ is $(x \cdot \sin\alpha - y \cdot \cos\alpha)^2$ the function $F(\alpha, S)$ that should be minimised in order to compute the orientation of S can be expressed as

$$\begin{aligned} F(\alpha, S) &= \iint_S \left(\left(x - \frac{m_{1,0}(S)}{m_{0,0}(S)}\right) \cdot \sin\alpha - \left(y - \frac{m_{0,1}(S)}{m_{0,0}(S)}\right) \cdot \cos\alpha\right)^2\ dx\ dy \\ &= \left(m_{2,0}(S) - \frac{(m_{1,0}(S))^2}{m_{0,0}(S)}\right) \cdot \sin^2\alpha\ +\ \left(m_{0,2}(S) - \frac{(m_{0,1}(S))^2}{m_{0,0}(S)}\right) \cdot \cos^2\alpha \\ &- \left(m_{1,1}(S) - \frac{m_{1,0}(S) \cdot m_{0,1}(S)}{m_{0,0}(S)}\right) \cdot \sin(2\alpha). \end{aligned} \tag{2}$$

Definition 1. *The orientation of a given shape S is defined by the angle α for which the function $F(\alpha, S)$ reaches the minimum.*

Elementary mathematics says that the angle α which defines the orientation of S (as given by Definition 1) satisfies the following equation:

$$\frac{\sin(2\alpha)}{\cos(2\alpha)} = \frac{2 \cdot \overline{m}_{1,1}(S)}{\overline{m}_{2,0}(S) - \overline{m}_{0,2}(S)}, \tag{3}$$

where $\overline{m}_{p,q}(S) = \iint\limits_S \left(x - \frac{m_{1,0}(S)}{m_{0,0}(S)}\right)^p \left(y - \frac{m_{1,0}(S)}{m_{0,0}(S)}\right)^q dxdy$ are centralised moments.

3 An Alternative Approach

In this section we start with a different motivation for the definition of shape orientation. It will turn out that this approach in its basic variant (when single component shapes are considered) leads to the same shape orientation as if computed based on the axis of the least moment of inertia, but the application to compound shapes leads to an essentially new method.

Let S be a given shape, and consider the line segments $[AB]$ whose end points A and B are points from S. Let $\overrightarrow{a}$ denote the unit vector in the direction α, i.e., $\overrightarrow{a} = (\cos\alpha, \sin\alpha)$. Also, let $\mathbf{pr}_{\overrightarrow{a}}[AB]$ be the projection of the line segment $[AB]$ onto a line that makes an angle α with the x-axis, while $|\mathbf{pr}_{\overrightarrow{a}}[AB]|$ denotes the length of such a projection. Then, it seems very natural to define the orientation of the shape S by the direction that maximises the integral $\iint\limits_{A\in S, B\in S}\iint |\mathbf{pr}_{\overrightarrow{a}}[AB]|^2 \; dx \; dy \; du \; dv$ of the squared length of projections of such edges onto a line having this direction. Thus we give the following definition.

Definition 2. *The orientation of a given shape S is defined by the angle α where the function*

$$G(\alpha, S) = \iint\iint\limits_{\substack{A=(x,y)\in S \\ B=(u,v)\in S}} |\mathbf{pr}_{\overrightarrow{a}}[AB]|^2 \; dx \; dy \; du \; dv \tag{4}$$

reaches its maximum.

Even though Definition 1 and Definition 2 come from different motivations it turns out that they are equivalent. Theorem 1 shows that the difference $G(\alpha, S) - 2 \cdot m_{0,0}(S) \cdot F(\alpha, S)$ depends only on the shape S but not on the angle α. Furthermore, this implies that the maximum of $G(\alpha, S)$ and minimum of $F(\alpha, S)$ are reached at the same point. In other words, the orientations computed by Definition 1 and Definition 2 are consistent.

Theorem 1. *The following equality holds:*

$$\begin{aligned} & G(\alpha, S) - 2 \cdot m_{0,0}(S) \cdot F(\alpha, S) \\ & = 2 \cdot ((m_{2,0}(S) + m_{0,2}(S)) \cdot m_{0,0}(S) - (m_{1,0}(S))^2 - (m_{0,1}(S))^2). \end{aligned} \tag{5}$$

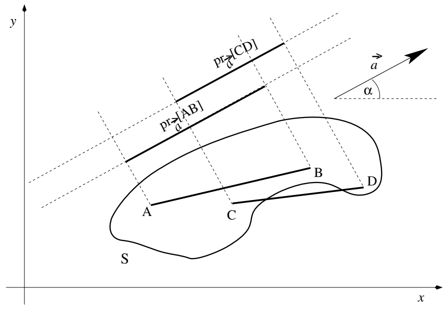

Fig. 1. Projections of all the line segments whose endpoints lie in S are considered, irrespective of whether the line segment intersects the boundary of S

Proof. Let $A = (x, y)$ and $B = (u, v)$. Then by using trivial equalities:

$$|\mathbf{pr}_{\overrightarrow{a}}[AB]|^2 = |(x-u, y-v)\cdot(\cos\alpha, \sin\alpha)|^2 \qquad \text{and}$$

$$\iiiint_{S\times S} x^p y^q u^r v^t \; dx\; dy\; du\; dv = m_{p,q}(S)\cdot m_{r,t}(S)$$

we complete the proof easily. Indeed,

$$\begin{aligned} & G(\alpha, S) - 2\cdot m_{0,0}(S)\cdot F(\alpha, S) \\ = & \iiiint_{S\times S} |\mathbf{pr}_{\overrightarrow{a}}[AB]|^2 \; dx\; dy\; du\; dv \;-\; 2\cdot m_{0,0}(S)\cdot F(\alpha, S) \\ = & \iiiint_{S\times S} ((x-u)\cdot\cos\alpha + (y-v)\cdot\sin\alpha)^2 \; dx\; dy\; du\; dv - 2\cdot m_{0,0}(S)\cdot F(\alpha, S) \\ = & \; 2(m_{2,0}(S) + m_{0,2}(S))\cdot m_{0,0}(S) - 2(m_{1,0}(S))^2 - 2(m_{0,1}(S))^2. \end{aligned}$$

□

4 Orientation of Compound Objects

Image analysis often deals with groups of shapes or with shapes that are composed of several parts. The desired properties of the computed orientation of such compound shapes can vary. Sometimes it is reasonable that the computed orientation is derived from the orientations of components of compound shape, whereas in other cases it is preferable that the orientation is a global property of the whole compound object. In this section we introduce a new definition for computing the orientation of such compound shapes and give some examples of computed orientations. The definition of such an orientation follows naturally from Definition 2.

Definition 3. *Let S be a compound object which consists of m disjoint shapes $S_1, S_2, \ldots, S_m$. Then the orientation of S is defined by the angle that maximises the function $G_{comp}(\alpha, S)$ defined by*

$$G_{comp}(\alpha, S) = \sum_{i=1}^{m} \iint\iint_{\substack{A=(x,y)\in S_i \\ B=(u,v)\in S_i}} |\mathbf{pr}_{\overrightarrow{a}}[AB]|^2 dx\ dy\ du\ dv. \tag{6}$$

The above definition allows an easy computation of the defined orientation.

Theorem 2. *The angle α where the function $G_{comp}(\alpha, S)$ reaches the maximum satisfies the following equation*

$$\frac{\sin(2\alpha)}{\cos(2\alpha)} = \frac{2 \cdot \sum_{i=1}^{m} (m_{1,1}(S_i) \cdot m_{0,0}(S_i) - m_{1,0}(S_i) \cdot m_{0,1}(S_i))}{\sum_{i=1}^{m} ((m_{2,0}(S_i) - m_{0,2}(S_i)) \cdot m_{0,0}(S_i) + (m_{0,1}(S_i))^2 - (m_{1,0}(S_i))^2)}$$

$$= \frac{2 \cdot \sum_{i=1}^{m} \overline{m}_{1,1}(S_i) \cdot m_{0,0}(S_i)}{\sum_{i=1}^{m} (\overline{m}_{2,0}(S_i) - \overline{m}_{0,2}(S_i)) \cdot m_{0,0}(S_i)}. \tag{7}$$

Proof. Similarly as in the proof of Theorem 1, setting $dG_{comp}(\alpha, S)/d\alpha = 0$ the proof follows easily. □

The following notes are given to point out some properties of $G_{comp}(\alpha, S)$.

Note 1. The computed orientation of a single component object (based on $F(\alpha, S)$ and $G(\alpha, S)$) breaks down when $\overline{m}_{1,1}(S) = \overline{m}_{2,0}(S) - \overline{m}_{0,2}(S) = 0$ because under these conditions $F(\alpha, S)$ and $G(\alpha, S)$ became constant functions (see (2) and (5)). Consequently no direction can be selected as a shape orientation. Analogously, when $\sum_{i=1}^{m} \overline{m}_{1,1}(S_i) = \sum_{i=1}^{m} (\overline{m}_{2,0}(S_i) - \overline{m}_{0,2}(S_i)) = 0$ holds then the orientation of the compound shape $S = S_1 \cup \ldots \cup S_m$ cannot be computed by $G_{comp}(\alpha, S)$.

Note 2. Any component S_i of a compound shape $S = S_1 \cup \ldots \cup S_m$ that is considered unorientable by $G(\alpha, S_i)$ (i.e. $G(\alpha, S_i) = const.$) will not contribute to (7), and is therefore ignored in the computation of $G_{comp}(\alpha, S)$. That is because $G(\alpha, S_i) = const.$ implies $\overline{m}_{1,1}(S_i) = 0$ and $\overline{m}_{2,0}(S_i) = \overline{m}_{0,2}(S_i)$.

Note 3. If all components S_i of a given shape S have identical orientation according to $G(\alpha, S_i)$ then this same orientation is also computed by $G_{comp}(\alpha, S)$.

Note 4. From (7) it can be seen that components of S contribute a weight proportional to $m_{0,0}(S_i)^3$.

The cubic weighting given to components in (7) seems excessive since it will tend to cause the larger components to strongly dominate the computed orientation. For instance, is a compound object S consists of a shape S_1 and the shape S_2' which is the dilation of a shape S_2 by a factor $\mathbf{r}$, i.e. $S_2' = \mathbf{r} \cdot S_2 = \{(\mathbf{r} \cdot x, \mathbf{r} \cdot y) \mid (x, y) \in S_2\}$ and, consequently, the size of S_2' increases when $\mathbf{r}$ increases too. Then, $m_{0,0}(S_2') = \mathbf{r}^2 \cdot m_{0,0}(S_2)$, $\overline{m}_{1,1}(S_2') = \mathbf{r}^6 \cdot \overline{m}_{1,1}(S_2)$, $\overline{m}_{2,0}(S_2') = \mathbf{r}^6 \cdot \overline{m}_{2,0}(S_2)$, $\overline{m}_{0,2}(S_2') = \mathbf{r}^6 \cdot \overline{m}_{0,2}(S_2)$. Entering the above estimates into (7) we obtain that $\sin(2\alpha)/\cos(2\alpha)$ equals

$$\frac{2 \cdot \overline{m}_{1,1}(S_1) \cdot m_{0,0}(S_1) + 2 \cdot \mathbf{r}^6 \cdot \overline{m}_{1,1}(S_2) \cdot m_{0,0}(S_2)}{(\overline{m}_{2,0}(S_1) - \overline{m}_{0,2}(S_1)) \cdot m_{0,0}(S_1) + \mathbf{r}^6 \cdot (\overline{m}_{2,0}(S_2) - \overline{m}_{0,2}(S_2)) \cdot m_{0,0}(S_2)}$$

and obviously the influence of S_2 to the computed orientation of S could be very big if the dilation factor $\mathbf{r}$ is much bigger than 1. This suggests a modification of (7) to enforce instead a linear weighting by area, namely:

$$\frac{\sin(2\alpha)}{\cos(2\alpha)} = \frac{2 \cdot \sum_{i=1}^{m} \overline{m}_{1,1}(S_i)/m_{0,0}(S_i)}{\sum_{i=1}^{m} (\overline{m}_{2,0}(S_i) - \overline{m}_{0,2}(S_i))/m_{0,0}(S_i)}. \quad (8)$$

If the orientation α of $S = S_1 \cup S_2' = S_1 \cup \mathbf{r} \cdot S_2$ is computed by (8) then $\sin(2\alpha)/\cos(2\alpha)$ equals $\frac{2 \cdot \overline{m}_{1,1}(S_1)/m_{0,0}(S_1) + 2 \cdot \mathbf{r}^2 \cdot \overline{m}_{1,1}(S_2)/m_{0,0}(S_2)}{(\overline{m}_{2,0}(S_1) - \overline{m}_{0,2}(S_1))/m_{0,0}(S_1) + \mathbf{r}^2 \cdot (\overline{m}_{2,0}(S_2) - \overline{m}_{0,2}(S_2))/m_{0,0}(S_2)}$. It is not difficult to imagine the situation where the size change of components of a compound objects should have no effect on the computed orientation. For instance, objects may be the same size in nature, but appear different sizes in the image due to varying distances from the camera. If we would like to avoid any impact of the size of the components on the computed orientation of a compound object then we can use the formula: $\frac{\sin(2\alpha)}{\cos(2\alpha)} = \frac{2 \cdot \sum_{i=1}^{m} \overline{m}_{1,1}(S_i)/(m_{0,0}(S_i))^2}{\sum_{i=1}^{m} (\overline{m}_{2,0}(S_i) - \overline{m}_{0,2}(S_i))/(m_{0,0}(S_i))^2}$.

Following on from Note 2, any component that is almost unorientable by $G(\alpha, S_i)$ will have little effect on the computation of $G_{comp}(\alpha, S)$. Thus we can say that $G_{comp}(\alpha, S)$ is not the same as computing a simple circular mean[1] of the orientations produced by $G(\alpha, S_i)$ since $G_{comp}(\alpha, S)$ weights the contributions of components according to both their area and their orientability.

The new methods (given by (7) and (8)) for computing orientation are demonstrated on some trademarks in figure 2. In figure 2a-c the computed orientations computed from $G_{comp}(\alpha, S)$ are different and preferable to those based on $F(\alpha, S)$ (in which the trade marks are considered as single component objects).

[1] The circular mean μ of a set of n orientation samples θ_i is defined as [8]:

$$\mu = \begin{cases} \mu' & \text{if } S > 0 \text{ and } C > 0 \\ \mu' + \pi & \text{if } C < 0 \\ \mu' + 2\pi & \text{if } S < 0 \text{ and } C > 0 \end{cases}$$

where $\mu' = \tan^{-1} \frac{S}{C}$, $C = \frac{1}{n} \sum_{i=1}^{n} \cos \theta_i$, $S = \frac{1}{n} \sum_{i=1}^{n} \sin \theta_i$.

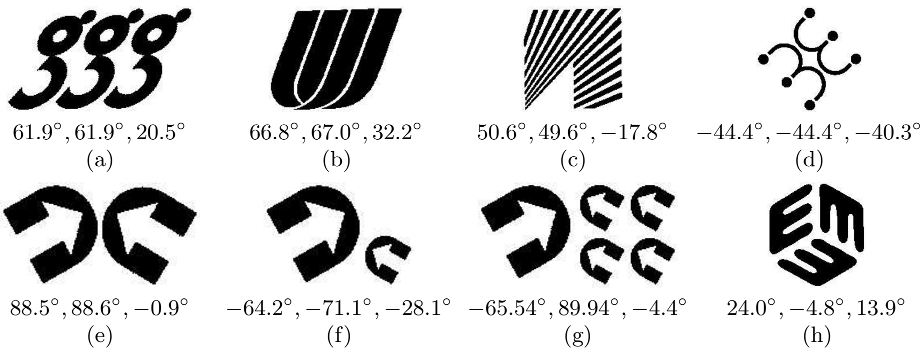

Fig. 2. Orientations of trademarks are shown computed by (7), (8), and (3)

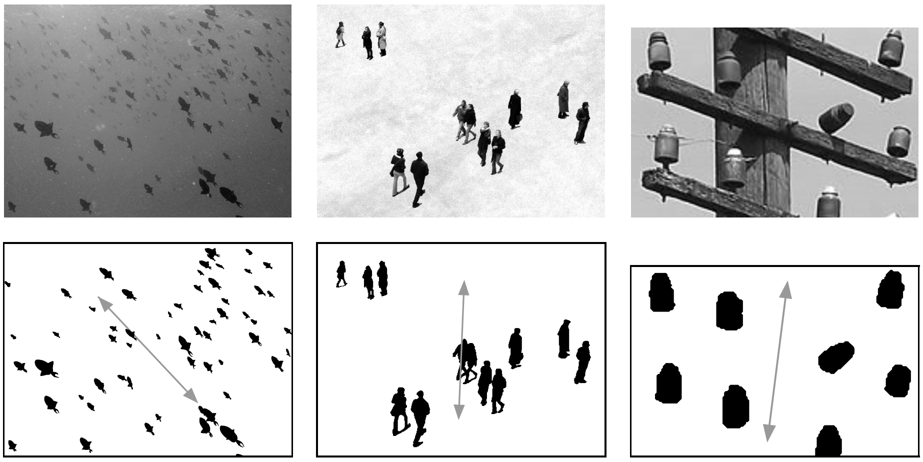

Fig. 3. Orientations computed by (8) of multiple components are overlaid

The computed orientations for the shape in figure 2d are very close and coincident with the one of the shape's symmetry axes. Some inconsistency is caused by the discretization process.

The trademark in figure 2e has reflective symmetry, and so the orientation given by $G_{comp}(\alpha, S)$ is along the symmetric axis as opposed to the standard method for estimating orientation. The individual orientations of the components are $\{-63.7°, 61.7°\}$. When one component is reduced in size (figure 2f) the larger component dominates (7) whereas this effect is reduced by (8). In figure 2g four quarter area components combine to have identical effect to a single full size component, and the orientation given by (8) is close to 90° again. The larger component still strongly dominates (7).

Figure 2h gives an example of a shape that is unorientable by (7) and (8) – see Note 1. The individual component orientations of $\{65.5°, -54.7°, 5.6°\}$ are well defined; $|\overline{m}_{2,0}(S_i) - \overline{m}_{0,2}(S_i)| + |m_{1,1}(S_i)|$ are around the value 400. However, $|\sum_{i=1}^{3} (\overline{m}_{2,0}(S_i) - \overline{m}_{0,2}(S_i))| + |\sum_{i=1}^{3} m_{1,1}(S_i)|$ is only equal to 6. This explains the inconsistency between the orientation estimates from (7) and (8) despite the component areas being almost equal.

The new method is further demonstrated on several natural scenes in figure 3. The overall orientations: 47.16°, 87.68°, and 82.87° computed by (8) are appropriate and seems to be more acceptable than orientations −15.6°, −8.1° and −4.3° computed by minimizing $F(\alpha, S)$. Note that in the last example the tilted blob has only a minor impact on the overall orientation estimated by (8).

5 Conclusion

In this paper we have considered an alternative approach for the computation of shape orientation. One benefit of such a new approach is that it leads to a new method for computing the orientation of compound objects which consist of several components. It was shown that such a defined compound shape orientation has several attractive properties. The computed orientations are given for several examples and they are reasonable. If applied to a single component objects the new defined shape orientation is consistent with the standard method for shape orientation computation, but the methods are different when applied to shapes consisting of several components. It is not possible to say that orientations computed by one of the methods are better then the orientations computed by the other one. A final judgement can be given only based on knowing the particular application where the methods are applied.

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