Measuring Shape: Ellipticity, Rectangularity, and Triangularity

Paul L. Rosin
Department of Computer Science
Cardiff University
UK
Paul.Rosin@cs.cf.ac.uk

Abstract

Object classification often operates by making decisions based on the values of several shape properties measured from the image. This paper describes and tests several algorithms for calculating ellipticity, rectangularity, and triangularity shape descriptors.

1. Introduction

A prevalent task in image analysis is the discrimination of objects based on their appearance. Various properties of appearance can be measured, falling into such categories as texture, colour, and shape. Shape is obviously a powerful tool for describing and differentiating objects, and has been extensively applied in many areas of computer vision.

Despite the introduction of powerful mathematical modelling techniques shape analysis remains problematic. One of the difficulties is that not all of the variations between shapes are necessarily significant. Minor perturbations caused by noise are an example and similarity transformations are also generally permissible. In addition, certain shapes are invariant to further transformations; for instance the aspect ratio of a rectangle can be modified without changing its intrinsic "rectangularity".

Given these difficulties, a popular approach is to design shape descriptors sensitive to specific aspects of shape such as eccentricity, Euler number, compactness, and convexity [10]. If a sufficiently varied set of descriptors are available then it should be possible to use some subset of them for shape classification and discrimination in a variety of tasks.

This paper investigates measuring three of the most basic shapes: ellipses, rectangles, and triangles. In computer vision circularity is extensively used as a shape measure, but there is little work on the measurement of the other shapes (but see [4, 6, 9]).

2. Ellipticity

2.1. Moment invariants (E_I)

The first approach is based on moment invariants. Since any ellipse can be obtained by applying an affine transform to a circle we use the simplest affine moment invariant [1] of the circle to characterise ellipses

$$I_1 = \frac{\mu_{20}\mu_{02} - \mu_{11}^2}{\mu_{00}^4}.$$

While all perfect circles will produce identical values of I_1 other shapes can also produce the same value. To help discriminate shape more precisely it would be possible to also incorporate higher order invariants. However, the disadvantage is that higher order moments are less reliable. In contrast I_1 only requires relatively low powers of second-order moments, and is thus more practical. The moments for the unit radius circle are

$$m_{pq} = \int_{-1}^{1} \int_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}} x^p y^q dy dx$$

allowing us to calculate the value of the invariant as $I_1 = \frac{1}{16\pi^2}$. The measure of ellipticity is then taken as

$$E_I = \begin{cases} 16\pi^2 I_1 & \text{if } I_1 \leq \frac{1}{16\pi^2} \\ \frac{1}{16\pi^2 I_1} & \text{otherwise} \end{cases}$$

which ranges over $[0, 1]$, peaking at 1 for a perfect ellipse.

2.2. Elliptic variance (E_V)

Peura and Iivarinen [4] described an "elliptic variance" which they used to measure ellipticity. The distances between the boundary points and their centroid are weighted by the covariance of the data points, and the variance Σ of this quantity calculated. For uniformity with the other measures we modify this to $E_V = \frac{1}{1+\Sigma}$.

2.3. Euclidean ellipticity (E_E)

A more direct and potentially more reliable approach is to robustly fit an ellipse to the region's boundaries and measure the Euclidean distances between the ellipse and region boundaries. The Least Median of Squares (LMedS) approach to feature fitting is taken [8].

In order to convert the error measure E into a useful ellipse measure it needs to be normalised such that it is scale invariant. It is therefore weighted by the square root of the original region's area A giving $E_E = \frac{1}{1+\frac{1}{N\sqrt{A}}E}$.

2.4. DFT (E_F)

Proffitt [6] describes an approach to measuring ellipticity and circularity based on the Discrete Fourier Transform (DFT). An ellipse is fitted to the shape by centering it on the region's centroid. The ellipse is then scaled such that its mean square of the lengths of the lines from the centroid to the boundary points matches the region's. Ellipticity is calculated (via the DFT) as the distance between corresponding points on the ellipse and the region.

3. Rectangularity

3.1. MBR (R_B)

The standard approach to measuring rectangularity is to use the ratio of the area of the region to the area of its minimum bounding rectangle (MBR).

3.2. Rectangular discrepancy (R'_D)

In an attempt to overcome the sensitivity of the MBR to noise Rosin [9] described an alternative in which a rectangle is fitted to the region based on its moments. Rectangularity is then measured as the normalised discrepancies between the areas of the rectangle and region. More precisely, given the following areas: R the difference between the rectangle and the region, D the difference between the region and the rectangle, and B the rectangle's area, then $R_D = 1 - \frac{R+D}{B}$. To overcome unreliable orientation estimates the method was considerably improved by considering both the original orientation estimate with and without a $45°$ offset [9]. The maximum of the two is retained as the final rectangularity measure R'_D.

3.3. Robust MBR (R_R)

Another approach to overcome the sensitivity of the MBR is to relax the requirement that the MBR must contain all the points. If the MBR need only contain the majority of the region then it should be more robust in the presence of small area deviations in the boundary. The formulation of the criterion for the MBR is similar to the previous measure, but modified to $\frac{R+D}{I}$ where the denominator I is the area of intersection of the region and the rectangle rather than the area of the rectangle. This expression provides a trade-off between forcing the rectangle to contain most of the data while keeping the rectangle as small as possible. The robust MBR is found by starting with the standard MBR rescaled to half its area. Powell's method [5] is then applied to iteratively minimise $\frac{R+D}{I}$ and the final fit provides the rectangularity as $R_R = 1 - \frac{R+D}{I}$.

4. Triangularity

4.1. Moment invariants (T_I)

The same approach can be used to characterise triangles by moment invariants as was done for ellipses. Any triangle can be considered as a simple right angled triangle aligned with the axes after an affine transformation is applied. The moments are

$$m_{pq} = \int_0^x \int_0^1 x^p y^q dx dy$$

which results in $I_1 = \frac{1}{108}$. The triangularity measure is thus

$$T_I = \begin{cases} 108 I_1 & \text{if } I_1 \leq \frac{1}{108} \\ \frac{1}{108 I_1} & \text{otherwise} \end{cases} .$$

4.2. Polygonal triangle approximation (T_A)

A second approach fits a geometric triangle model to the data, and measures the error between the model and data. Fitting is performed by finding a polygonal approximation of the boundary. Dynamic programming is used to find the optimal three line polygon approximation minimising $E = \sum_i d_i$, the summed L_1 error, where d_i is the shortest Euclidean distance from $\boldsymbol{p}_i$ to the triangle [7]. Since this is computationally expensive the data is first subsampled by a factor of five. The final triangularity measure is $T_A = \frac{1}{1+\frac{E}{A}}$ where A is the area of the region.

4.3. Projections (T_P)

A computationally more efficient approach to fitting the triangle model is to work with the region's projections. Successive applications of projections enables the region's shape to be continually simplied. As shown in fig. 1 first the horizontal and vertical projections are found, and then each of these are further projected. For any region which is a

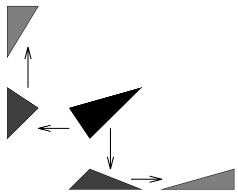

Figure 1. Horizontal and vertical projections.

triangle of arbitrary shape and position the resulting two final projections will both be right angled, isothetic triangles. These are much simpler to analyse than the initial arbitrary triangle. Their parameters can be estimated from their centroids using the relation $(x_c, y_c) = \frac{1}{3}(b, a)$ where a and b are the lengths of the sides. For non-triangular regions the triangles fitted to their projections will be in error. If E_1 and E_2 are the summed errors between the fits and the projections, then the triangularity measure is $T_P = 1 - \frac{E_1+E_2}{2A}$.

4.4. Minimum bounding triangle (T_B)

Another alternative approach to fitting a triangle is to use the region's minimum bounding triangle (MBT). O'Rourke *et al.* [3] describe an optimal $O(n)$ algorithm to determine the MBT. Analogous to the standard approach to measuring rectangularity, triangularity is calculated as the ratio of the area of the region to the area of its MBT.

5. Evaluation

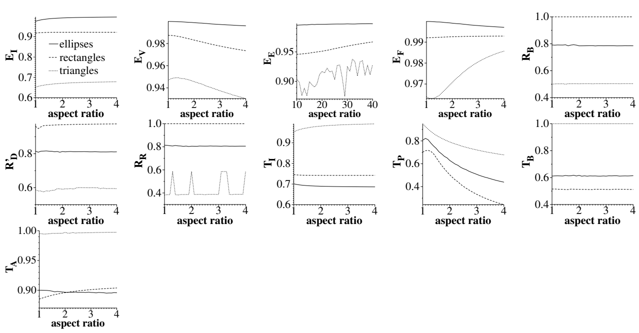

Figure 2. Application to ellipses, rectangles, and triangles at different aspect ratios.

As a first test the shape descriptors are applied to perfect synthetic images containing ellipses, rectangles and triangles with different aspect ratios (fig. 2). They all successfully discriminate between the three shapes with the exception of T_A. While triangles receive the top score as expected, the ranking of the measured triangularity for ellipses and rectangles varies. A more significant defect is apparent in T_P which is not invariant over aspect ratio.

Figure 3. An ellipse, rectangle, and triangle at the highest noise level.

The second experiment tests the robustness of the measures to noise. For a fixed aspect ratio of 1.6 increasing amounts of uniform noise were added to the shapes' boundaries (see fig. 3).. In terms of discriminating the target shape from the other two shapes most methods still perform well (see fig. 4). Only E_F breaks down, although E_V also seems potentially sensitive.

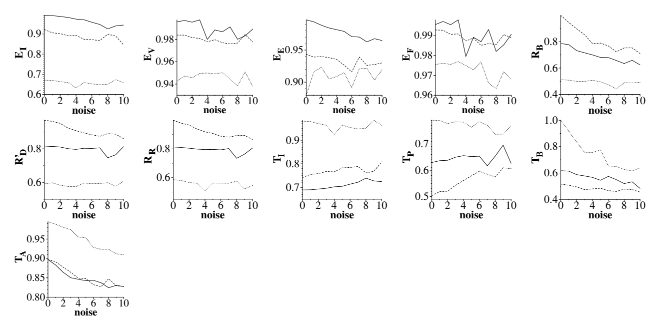

Figure 4. Application to ellipses, rectangles, and triangles at different noise levels.

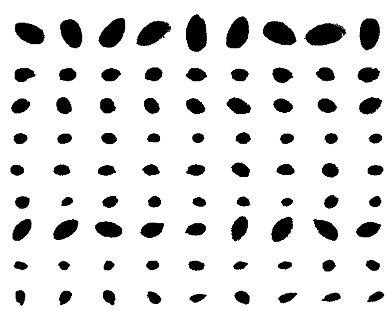

Figure 5. Examples of data; each row contains one type of seed or bean.

The third evaluation applies the measures to real data. Fig. 5 shows a variety of seeds and beans to be distinguished. While they are all roughly circular or oval the similarity in shape over the classes is evident and, in combination with the noise and the variability in shape aspect ratio within classes, makes the discrimination task based on shape alone difficult. The data set of 260 samples was split into equal training and testing parts, and classification was performed using Murthy *et al.*'s oblique decision trees [2] applied to two measures at a time. In addition to the ellipticity, rectangularity, and triangularity measures two sets of moment invariants were considered (those invariant to similarity transforms as well as affine invariants), and the standard shape descriptors of eccentricity, circularity, compactness, and convexity [10]. The classification accuracies are given in tables 1-4. The best result is achieved by the combination of E_F and T_I, and the second best result by E_I and R_R. The similarity transform moment invariants also perform well, while the affine moment invariants and the standard shape descriptors do significantly worse.

	R_B	R'_D	R_R	T_I	T_P	T_B	T_A
E_I	37.69	33.85	42.31	30.77	34.62	26.92	33.08
E_V	17.69	41.54	20.77	40.00	18.46	15.38	25.38
E_U	19.23	38.46	25.38	36.15	27.69	18.46	22.31
E_F	20.77	33.85	19.23	43.08	30.00	20.00	26.92
R_B	–	–	–	40.00	19.23	13.08	26.92
R'_D	–	–	–	29.23	36.92	30.00	26.15
R_R	–	–	–	38.46	16.92	17.69	23.08

Table 1. Classification accuracy using ellipticity, rectangularity, and triangularity.

	ϕ_2	ϕ_3	ϕ_4
ϕ_1	26.92	35.38	36.15
ϕ_2	–	40.00	40.77
ϕ_3	–	–	31.54

Table 2. Classification accuracy using similarity transform moment invariants.

	I_2	I_3
I_1	21.54	25.38
I_2	–	19.23

Table 3. Classification accuracy using affine moment invariants.

6. Conclusions

In this paper an extensive list of algorithms for implementing ellipticity, rectangularity, and triangularity shape descriptors is provided. Ideally they should all be invariant to scalings, rotations, and rotations, and additionally stretching along the axes in the case of rectangles and more generally affine deformations in the cases of ellipses and triangles. Moreover, they should be robust in the presence of noise. Tests showed that, with the exception of T_P, the first requirement was reasonably satisfied by all the descriptors (with respect to the target shape). Again, according to the second requirement, most of the descriptors were satisfactory apart from E_F and possibly E_V.

Further testing was provided by a classification task involving images of nine types of seeds and beans. Compared to the common, standard shape descriptors also tested the combination of E_F and T_I, and also E_I and R_R achieved better classification accuracies. However, it should be remembered that the performance of individual shape descriptors is necessarily application dependent. Thus it is too early to give any general recommendations regarding the comparative merits of the descriptors except for the above caveats concerning E_V, E_F, and T_P.

	circularity	compactness	convexity
eccentricity	30.00	16.92	35.38
circularity	–	29.23	23.08
compactness	–	–	29.23

Table 4. Classification accuracy using standard shape descriptors.

References

[1] J. Flusser and T. Suk. Pattern recognition by affine moment invariants. *Pattern Recognition*, 26:167–174, 1993.

[2] S.K. Murthy, S. Kasif, and S. Salzberg. System for induction of oblique decision trees. *Journal of AI Research*, 2:1–33, 1994.

[3] J. O'Rourke, A. Aggarwal, S. Maddila, and M. Baldwin. An optimal algorithm for finding minimal enclosing triangles. *J. Algorithms*, 7:258–269, 1986.

[4] M. Peura and J. Iivarinen. Efficiency of simple shape descriptors. In *Aspects of Visual Form*, pages 443–451. World Scientific, 1997.

[5] W.H. Press, B.P. Flannery, S.A. Teukolsky, and W.T. Vettering. *Numerical Recipes in C*. Cambridge University Press, 1990.

[6] D. Proffitt. The measurement of circularity and ellipticity on a digital grid. *Pattern Recognition*, 15(5):383–387, 1982.

[7] P.L. Rosin. Techniques for assessing polygonal approximations of curves. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 19(6):659–666, 1997.

[8] P.L. Rosin. Further five-point fit ellipse fitting. *CVGIP: Graphical Models and Image Processing*, 61(5):245–259, 1999.

[9] P.L. Rosin. Measuring rectangularity. *Machine Vision and Applications*, 11:191–196, 1999.

[10] M. Sonka, V. Hlavac, and R. Boyle. *Image Processing, Analysis, and Machine Vision*. Chapman and Hall, 1993.