

Convexity Measure for Shapes with Partially Extracted Boundaries

Joviša Žunić,^{a, *} Paul L. Rosin^b

^a Department of Computer Science, University of Exeter, Exeter EX4 4QF, U.K.
e-mail: J.Zunic@ex.ac.uk

^b School of Computer Science, Cardiff University, Cardiff CF24 3AA, Wales, U.K.
e-mail: Paul.Rosin@cs.cf.ac.uk

Abstract

Shape descriptors are used in many computer vision tasks. Convexity is one of the most widely used shape descriptors in practical applications and also one of the most studied in the literature. There are already several defined convexity measures. The most standard one comes from the comparison between a given shape and its convex hull, but there also other approaches. Independently of whether the convexity descriptors are area based or boundary based, all of them assume that the shape (or shape boundary) is completely known and that the measures apply to the complete data. In this paper we define a convexity measure that can be applied to shapes with partially extracted boundaries. More formally, the new convexity measure deals with planar curves or with disconnected curves consisting of several planar curve segments.

Key words: Shape, shape descriptors, convexity, measure.

*J. Žunić is also with the Mathematical Institute, Serbian Academy of Arts and Sciences, Belgrade.

1 Introduction

Shape is one of the basic terms in the area of computer vision and image processing. Usually a shape corresponds to a compact planar region. There are many ways to characterise shapes; one approach is to use a suitably selected set of shape descriptors. One of the mostly employed shape descriptors is *convexity*. Over the years many convexity measures have been developed (e.g. [1, 2, 3, 4, 6, 7]) and have been applied to tasks such as image segmentation, shape decomposition, object classification, grouping, etc. Their usefulness in such applications has led to a continual interest in shape descriptors and development of new ones.

Typically, approaches to defining and computing global shape descriptors assume that the complete shape (or the shape boundary) is known. However, in some image processing tasks it is not possible to extract the complete shape boundary. For example, if there is some overlap between objects or there is no clear difference between foreground and background pixels (see Fig. 1). In such cases only fragments of the boundary can be extracted; nevertheless, it would still be worth computing shape information from the available data. Another motivation for this work is that some (very thin) objects, because of their nature, are more adequately represented by curve segments than by planar regions. A person’s signature is an example. Based on the previous observation it seems reasonable to define descriptors that correspond to a curve or a disconnected curve consisting of several curve segments.

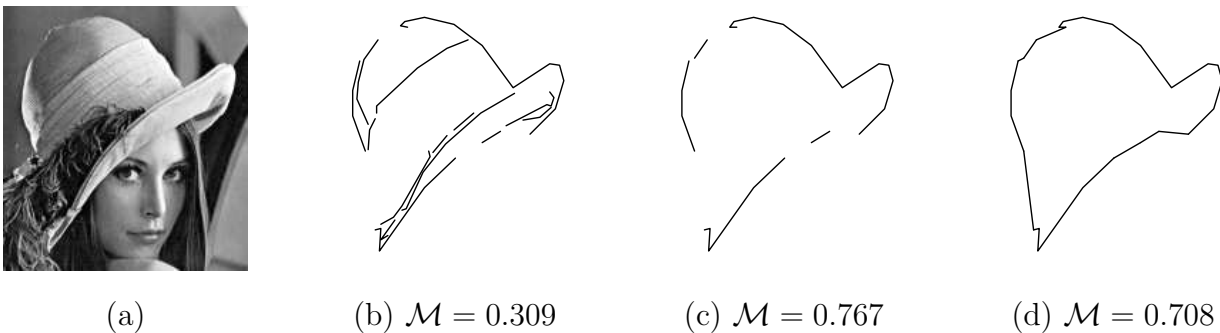


Figure 1: (a) Extracting the boundary of **Lena**’s hat is difficult due to poor contrast in places as well as clutter. (b) After edge detection and linking, the edge segments relating to the hat have been manually selected. (c) The outer boundary curves. (d) Gaps between the outer boundary curves completed by straight line segments. The new convexity measure proposed in this paper can cope with all those situations.

2 Convexity Measure for Curves

In this section we define a convexity measure for curves or disconnected curves consisting of several curve segments.

Perhaps the most common definition of a convex curve is the next definition that comes from the mathematical theory.

Definition 1 *A curve γ is convex if and only if for each point A on the curve γ there is a line l such that the curve γ completely lies in one of the halfplanes determined by the line l .*

For our purpose we will use another definition that is equivalent to Definition 1.

Definition 2 *A curve γ is convex if and only if for each two points A and B on the curve γ the open line segment (AB) does not intersect the curve γ (i.e. $(AB) \cap \gamma = \emptyset$) or (AB) completely belongs to the curve γ (i.e. $(AB) \subset \gamma$).*

Remark. In mathematical terms, the *strict convexity* of a given curve γ does not allow straight line sections in γ . We will use the convexity definition as given by Definition 2 rather than the strict convexity definition. In computer vision and image processing applications we often use polygonal approximations, and such a weaker definition allows polygons which are boundaries of convex polygonal regions to be understood as convex curves.

For our purpose we need a straightforward extension of Definition 2 to the curves that are not necessarily connected. In other words, the curve γ from Definition 2 can be a disconnected curve $\gamma = \gamma_1 \cup \dots \cup \gamma_n$ consisting of n curve segments $\gamma_1, \dots, \gamma_n$.

Now, based on Definition 2, we define a new convexity measure for curves and for disconnected curves consisting of several curve segments.

Definition 3 *Let $\gamma = \gamma_1 \cup \dots \cup \gamma_n$ be a curve that consists of $n \geq 1$ curve segments, and let A and B be two randomly selected points from γ . The convexity measure $\mathcal{M}(\gamma)$ is defined as the probability that*

- *the open straight line segment (AB) does not intersect γ (i.e. $(AB) \cap \gamma = \emptyset$), or*
- *the open straight line segment (AB) completely belongs to γ (i.e. $(AB) \subset \gamma$).*

The following theorem summarises the desirable properties of the convexity measure proposed here.

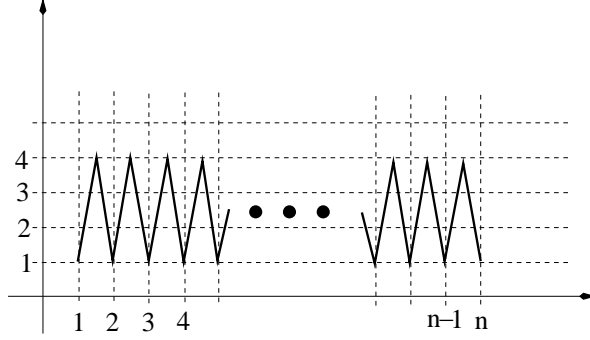


Figure 2: The measured convexity of the presented zigzag curve tends to 0 as n tends to ∞ .

Theorem 1 *Let $\gamma = \gamma_1 \cup \dots \cup \gamma_n$ be a (not necessarily connected) curve. Then*

- i) $\mathcal{M}(\gamma) \in (0, 1]$;*
- ii) $\mathcal{M}(\gamma) = 1$ if and only if there is a convex curve ρ such that $\gamma \subset \rho$;*
- iii) for any $\varepsilon > 0$ there is a curve γ such that $\mathcal{M}(\gamma) < \varepsilon$;*
- iv) $\mathcal{M}(\gamma)$ is invariant under similarity transformations.*

Proof. The items *i)*, *ii)*, and *iv)* follow easily from the definition.

In order to prove *iii)* just consider the “zigzag” curve from Fig.2 which consists of $2(n-1)$ edges having the length $a = \sqrt{37}/2$. By using the total probability formula we derive ($P(E|F)$ means “probability of E given F ”):

$$\begin{aligned}
\mathcal{M}(\gamma) &= P((AB) \cap \gamma = \emptyset \text{ or } P((AB) \subset \gamma \mid A \text{ belongs to the first edge of } \gamma) \\
&+ P((AB) \cap \gamma = \emptyset \text{ or } P((AB) \subset \gamma \mid A \text{ belongs to the last edge of } \gamma) \\
&+ P((AB) \cap \gamma = \emptyset \text{ or } P((AB) \subset \gamma \mid A \text{ belongs to a middle edge of } \gamma) \\
&= \frac{1}{2(n-1)} \cdot \frac{2}{2(n-1)} + \frac{1}{2(n-1)} \cdot \frac{2}{2(n-1)} + \sum_{i=1}^{2(n-2)} \frac{1}{2(n-1)} \cdot \frac{3}{2(n-1)} = \frac{3n-4}{2(n-1)^2}.
\end{aligned}$$

Since $\mathcal{M}(\gamma) = \frac{3n-4}{2(n-1)^2} \rightarrow 0$ as $n \rightarrow \infty$, the statement holds. \square

3 Computation of $\mathcal{M}(\gamma)$

The exact value of $\mathcal{M}(\gamma)$ can be computed only in particular cases as it was in case of the zigzag curve from Fig.2 (see the proof of Theorem 1). In most image processing tasks

the equation of γ or the equations of γ -segments remain unknown. In such a case it is only possible to estimate the convexity $\mathcal{M}(\gamma)$. We give the following simple and natural procedure that can be used for an estimate of $\mathcal{M}(\gamma)$.

Estimation of $\mathcal{M}(\gamma)$

Input: A curve $\gamma = \gamma_1 \cup \dots \cup \gamma_n$ and a number M that depends on the required precision that should be reached.

Step 1 Estimate the total length of curve γ :

$$\text{length_of_}\gamma = \text{length_of_}\gamma_1 + \dots + \text{length_of_}\gamma_n$$

and express the interval $[0, \text{length_of_}\gamma]$ as the union of n intervals

$$[0, \text{length_of_}\gamma] = I_1 \cup I_2 \cup \dots \cup I_n =$$

$$[0, \text{length_of_}\gamma_1) \cup [\text{length_of_}\gamma_1, \text{length_of_}\gamma_1 + \text{length_of_}\gamma_2)$$

$$\cup \dots \cup [\sum_{1 \leq i \leq n-1} \text{length_of_}\gamma_i, \text{length_of_}\gamma];$$

Step 2 Generate M pairs

$$(a_1, b_1), (a_2, b_2), \dots, (a_M, b_M)$$

of random numbers from the interval $[0, \text{length_of_}\gamma]$;

(a) For any pair (a_i, b_i) , $i = 1, 2, \dots, N$, determine the intervals $I_{u(a_i)}$ and $I_{v(b_i)}$ such that $a_i \in I_{u(a_i)}$ and $b_i \in I_{v(b_i)}$;

(b) Determine the point X_i that is on distance $a_i - \sum_{1 \leq i \leq u(a_i)-1} \text{length_of_}\gamma_i$ from the starting point of $\gamma_{u(a_i)}$ and determine the point Y_i that is on distance $b_i - \sum_{1 \leq i \leq v(b_i)-1} \text{length_of_}\gamma_i$ from the starting point of $\gamma_{v(b_i)}$;

(c) Check if the straight line segment $(X_i Y_i)$ intersects γ or belongs to it.

Output: The convexity $\mathcal{M}(\gamma)$ is estimated as $\frac{K}{M}$ where K is the number of straight line segments $(X_i Y_i)$ which do not intersect γ or completely belong to it.

4 Experimental results

In this section, the convexity measure is demonstrated on several examples. In all cases the data was preprocessed by performing polygonal approximation using Ramer's algorithm [5]

with a threshold of 2, and 100000 line segment tests were used to compute \mathcal{M} . Fig 3 shows handwritten digits which contain substantial natural variations, not only in their overall shape, but also in topology.

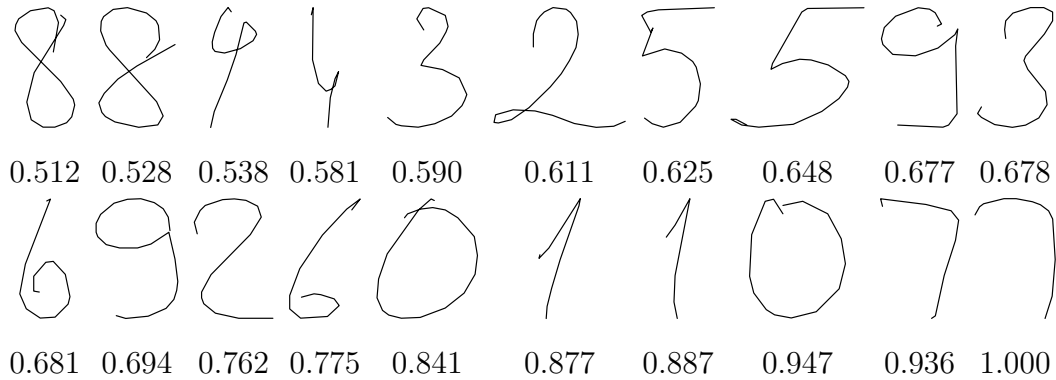


Figure 3: Handwritten digits ordered by their \mathcal{M} convexity values.

For instance, of each of the “0”, “2” and “8” digits one example is self intersecting while the other is not. Nevertheless, ranking the digits according to convexity demonstrates a reasonable separation between many of them (i.e. “1”, “4”, “5”, “7” and “8”), indicating that the convexity measure would be a useful property for classification of the digits.

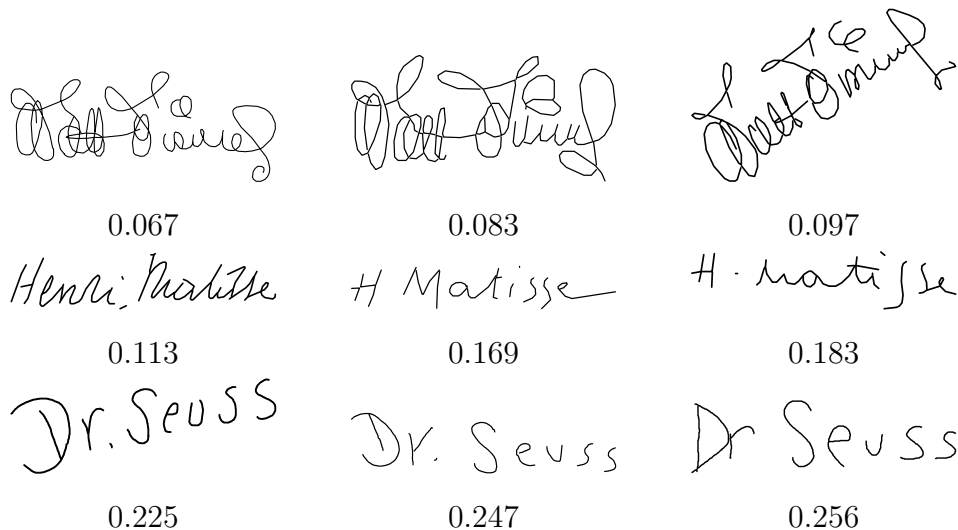


Figure 4: Signatures of Walt Disney, Henri Matisse and Dr. Seuss ordered by their \mathcal{M} convexity values.

In the second example signatures, which are made up from multiple curve segments, are used (see Fig 4). Again noticeable variability is evident within each individual. Note that since these curves are more complex than the individual digits in Fig 3 they have considerably lower convexity values. It can be seen from the ranking by \mathcal{M} that convexity is a sufficient descriptor for classification in this small example.

References

- [1] L. Boxer. Computing deviations from convexity in polygons. *Pattern Recognition Letters*, 14:163–167, 1993.
- [2] R. Kakarala. Testing for convexity with Fourier descriptors. *Electronics Letters*, 34(14):1392–1393, 1998.
- [3] R.R. Martin and P.L. Rosin. Turning shape decision problems into measures. *Int. J. Shape Modelling*, 10(1):83–113, 2004.
- [4] E. Rahtu, M. Salo, and J. Heikkilä. A new convexity measure based on a probabilistic interpretation of images. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 28(9):1501–1512, 2006.
- [5] U. Ramer. An iterative procedure for the polygonal approximation of plane curves. *Computer Graphics and Image Processing*, 1:244–256, 1972.
- [6] P.L. Rosin and C.L. Mumford. A symmetric convexity measure. *Computer Vision and Image Understanding*, 103(2):101–111, 2006.
- [7] J. Žunić and P.L. Rosin. A new convexity measurement for polygons. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 26(7):923–934, 2004.