# Structure and enumeration of toroidal and projective-planar graphs with no $K_{3,3}$'s

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By Kuratowski's theorem, a graph $G$ is non-planar if and only if it contains a subdivision of $K_5$ or $K_{3,3}$. A graph $G$ does not contain a $K_{3,3}$-subdivision if and only if it does not contain a $K_{3,3}$-minor. Therefore such a graph is called a graph *with no* $K_{3,3}$*'s*. The graphs with no $K_{3,3}$'s can be described recursively in terms of $K_5$'s and 2-connected planar graphs.

We provide structure theorems for toroidal and projective-planar graphs with no $K_{3,3}$'s in terms of 2-pole planar networks substituted for the edges of canonically defined non-planar graphs. These non-planar graphs are respectively called *toroidal cores* and *projective-planar cores*. The decompositions imply algorithms to detect toroidal and projective-planar graphs with no $K_{3,3}$'s. The algorithms can be implemented to run in linear time.

A proper use of mixed generating functions with an edge counter is described in detail for the operation of substitution of 2-pole networks into the edges of a graph. As a result, we count labelled 2-connected toroidal and projective-planar graphs with no $K_{3,3}$'s, and labelled 2-connected homeomorphically irreducible planar, toroidal and projective-planar graphs with no $K_{3,3}$'s. We are currently working on the unlabelled enumeration of these graphs, and have already counted the isomorphism classes of toroidal cores.