CM3106 Chapter 8: Multimedia Data Graphics, Images and Video

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Graphic/Image File Formats

Common graphics and image file formats:

- *http://www.dcs.ed.ac.uk/home/mxr/gfx/* — comprehensive listing of various formats.
- See *Encyclopedia of Graphics File Formats* book in library
- Most formats incorporate *compression*, including lossless or lossy
- Graphics, video and audio compression techniques in next Chapter.

Graphic/Image Data Structures

"A picture is worth a thousand words, but it uses up three thousand times the memory."

Image Format:

- A digital image consists of many picture elements, termed **pixels**.
- The number of pixels determine the quality of the image (**resolution**).
- Higher resolution always yields better quality.
- A *bit-map* representation stores the graphic/image data in the same manner that the computer monitor contents are stored in video memory.

Bit-Map (Black-and-White) Images

- Each pixel is stored as a single bit (0 or 1)
- A 640 x 480 bit-mapped image requires 37.5 KB of storage.
- **Dithering** is often used for displaying monochrome images

Gray-scale Images

- Each pixel is usually stored as a byte (value between 0 to 255)
- A dark pixel may have a value of 10; a bright one may be 240
- A 640 x 480 greyscale image requires over 300 KB of storage.

Dithering

- Dithering is often used when converting greyscale images to bit-mapped ones e.g. for printing
- The main strategy is to replace a pixel value (from 0 to 255) by a larger pattern (e.g. 4×4) such that the number of printed dots approximates the greyscale level of the original image
- If a pixel is replaced by a 4×4 array of dots, the intensities it can approximate from 0 (no dots) to 16 (full dots).
- Given a 4×4 dither matrix e.g.

$$\begin{pmatrix} 0 & 8 & 2 & 10 \\ 12 & 4 & 14 & 6 \\ 3 & 11 & 1 & 9 \\ 15 & 7 & 13 & 5 \end{pmatrix}$$

we can re-map pixel values from 0–255 to a new range 0–16 by dividing the value by (256/17) (and rounding down).

Dithering (cont.)

A simple dithering approach:

- Replace each pixel by a 4×4 dots (binary pixels). If the remapped intensity is $>$ the dither matrix entry, put a dot at the position (set to 1) otherwise set to 0.
- Note that the size of the dithered image may be much larger. Since each pixel is replaced by 4×4 array of dots, the image becomes 16 times as large.
- To keep the image size: an ordered dither produces an output pixel with value 1 **iff** the remapped intensity level **just at the pixel position** is greater than the corresponding matrix entry.

24-bit Colour Images

- Each pixel is represented by three bytes (e.g., RGB)
- Supports 256 x 256 x 256 (16,777,216) possible colours
- A 640 x 480 24-bit colour image is 921.6 KB large
- Some colour images are 32-bit images,
  - the extra byte of data for each pixel is used to store an **alpha** value representing special effect information

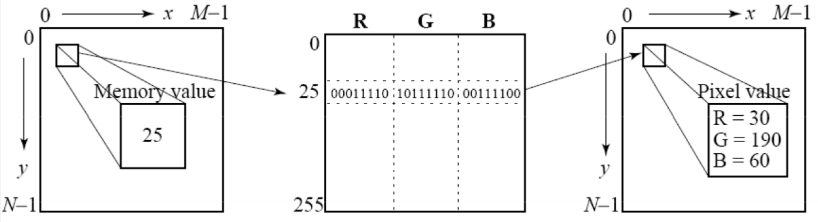
8-bit Colour Images

- One byte for each pixel
- Supports 256 out of the millions possible
- Acceptable colour quality
- Requires Colour Look-Up Tables (LUTs)
- A 640 x 480 8-bit colour image takes up 307.2 KB (**cf**. **8-bit greyscale**)

Colour Look-Up Tables (LUTs)

The Colour LUT

- Store only the index of the colour LUT for each pixel
- Look up the table to find the colour (RGB) for the index
- LUT needs to be built when converting 24-bit colour images to 8-bit: grouping similar colours (each group assigned a colour entry)
- Possible for palette animation by changing the colour map



Standard System Independent Formats

GIF (GIF87a, GIF89a)

- Graphics Interchange Format (GIF) devised by the UNISYS Corp. and Compuserve, initially for transmitting graphical images over phone lines via modems
- Uses the Lempel-Ziv Welch algorithm (a form of Huffman Coding), modified slightly for image scan line packets (line grouping of pixels) so lossless — **Algorithm Soon**
- Limited to only 8-bit (256) colour images, suitable for images with few distinctive colours (e.g., graphics drawing)
- Supports *interlacing*
- GIF89a: supports simple animation, transparency index etc.

JPEG

JPEG Standard:

- A standard for photographic image compression created by the Joint Photographic Experts Group
- Takes advantage of limitations in the human vision system to achieve high rates of compression
- Lossy compression which allows user to set the desired level of quality/compression
- **Algorithm Soon** — Detailed discussions in next chapter on compression.

TIFF

Tagged Image File Format (TIFF)

- TIFF, stores many different types of images (e.g., bit-map, greyscale, 8-bit & 24-bit RGB, etc.) –> **tagged**
- Developed by the Aldus Corp. in the 1980's and later supported by the Microsoft
- TIFF is a typically lossless format
  - Can utilise JPEG tag which allows for JPEG compression
- It does not provide any major advantages over JPEG and is not as user-controllable it appears to be declining in popularity.

PNG

Portable Network Graphics (PNG)

- PNG meant to supersede GIF standard
- Features of PNG
  - Support up to 48 bits per pixel – more accurate colours
  - Support description of gamma-correction and alpha-channel for controls such as transparency
  - Support progress display through 8×8 blocks.

Postscript/Encapsulated Postscript

Postscript (PS)/Encapsulated Postscript (EPS)

- A typesetting language which includes text as well as **vector**/structured **graphics** and **bit-mapped images**
- Output in several popular graphics programs (*E.g.* Illustrator, FreeHand)
- Does not provide compression, files are often large
  - Although able to link to external compression applications

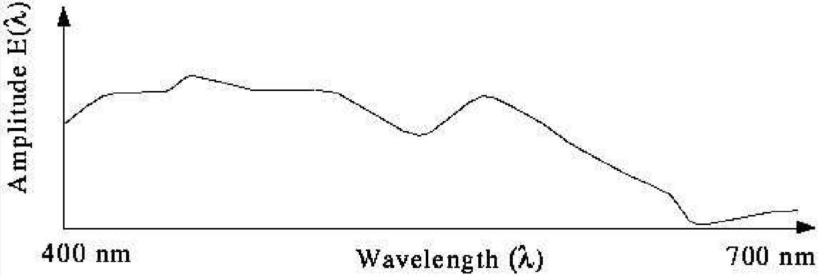
System Dependent Format

BMP (bitmap image file or device independent bitmap (DIB) file format)

- A system standard graphics file format for Microsoft Windows
- Raster image format
- Used in Many PC Graphics programs, Cross-platform support
- It is capable of storing 24-bit bitmap images

Light and Spectra

- Visible light is an **electromagnetic wave** in the **400nm - 700 nm** range.
- Most light we see is not one wavelength, it's a combination of many wavelengths (**cf.** a Rainbow).

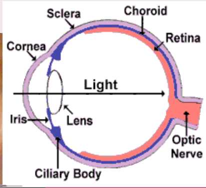


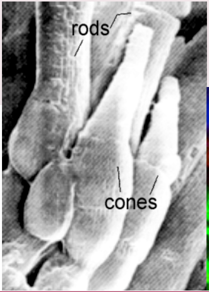
- The profile above is called a **spectra**.

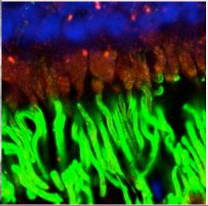
The Human Eye

Basic Eye Anatomy

- The eye is basically similar to a camera
- It has a lens to focus light onto the Retina of eye
- Retina full of **neurons**
- Each neuron is either a *rod* or a *cone*.
- Rods are not sensitive to colour.



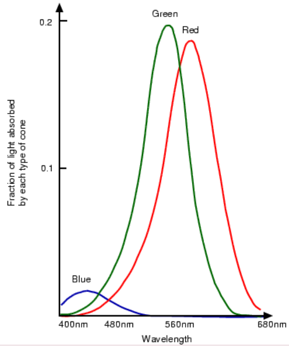




Cones and Perception

How Humans Perceive Colour:

- Cones come in **3** types: **red**, **green** and **blue**.
- Each responds differently to various frequencies of light. The following figure shows the spectral-response functions of the cones and the **luminous-efficiency** function of the eye.



Luminous-efficiency Function of the Human Eye's Cones

RGB Colour Space

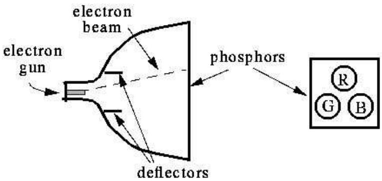
- Colour Space is made up of Red, Green and Blue intensity components

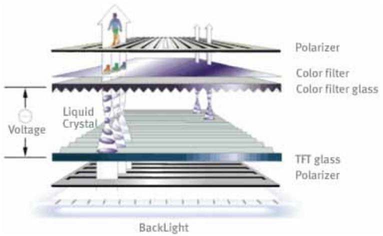
RGB Colour Space

Red, **Green**, **Blue** (RGB) Respective Intensities

Colour Displays

- (Old) CRT displays have three phosphors (RGB) which produce a mix of colours when excited with electrons.
- (Newer) Colour LCD panels (typically thin-film-transistor liquid-crystal displays (TFT LCD)): Transistor switch for each (R, G or B) pixel





- The **gamut** of colours is all colours that can be reproduced using the three primaries
- The gamut of a colour monitor is smaller than that of color models, E.g. CIE (LAB) Model — **see later**.

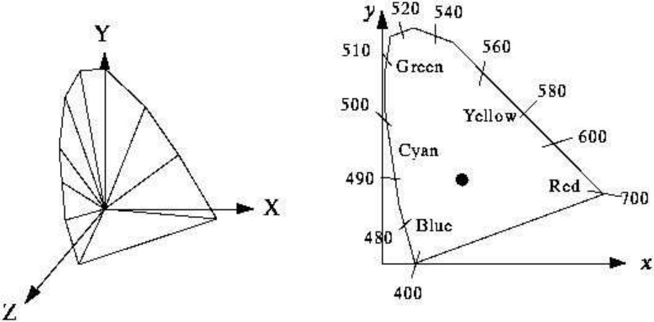
CIE Colour Model

Human focussed Colour Models:

- In 1931, the CIE defined three standard primaries **(X, Y, Z)** .
  - The **Y** primary was intentionally chosen to be identical to the **luminous-efficiency** function of **human eyes** (**Perceptual Model**).
- All visible colours are in a *horseshoe* shaped cone in the X-Y-Z space. Consider the plane $X+Y+Z=1$ and project it onto the X-Y plane, we get the *CIE chromaticity diagram*.

CIE Chromaticity Diagram

CIE chromaticity diagram:

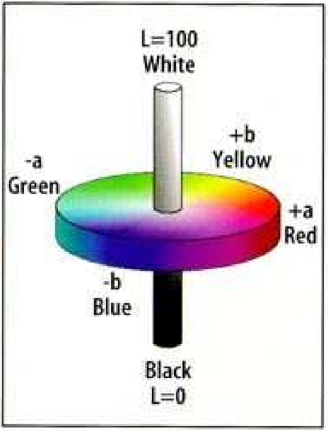


- The edges represent the **pure** colours
- **White** (a blackbody radiating at 6447 kelvin) is at the **dot**
- **Vector-based colour model**: When **added**, any two colours (points on the CIE diagram) produce a point on the line between them.

L*a*b* (Lab) Colour Model

A refined CIE model:

- Named CIE **L*a*b*** in 1976
- **Luminance**:**L**
  Chrominance:
  - **a** – ranges from green to red,
  - **b** – ranges from blue to yellow
- Used by *Photoshop*



Lab model

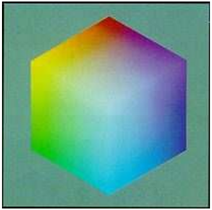
Lab Image Space

Original Color Image

L, A, B Image Intensities

Colour Image and Video Representations

- **Recap**: A grey-scale image is a 2-D array of integers.
- **Recap**: A **true colour image**: a 2-D array of (R,G,B) integer triplets. These triplets encode how much the corresponding colour pixel should be excited in devices such as an LCD monitor.



- **Recap**: Use **Luminance** and **Chrominance** to **better encode** how Humans **'see'** colour.

Besides the **RGB** representation, **YIQ** and **YUV** are the two commonly used in video.
(Latter two use a model of **Luminance** and **Chrominance**)

YIQ Colour Model

YIQ Colour Model

- YIQ is used in colour TV broadcasting, it is downward compatible with B/W TV.
- Y (luminance) is the CIE Y primary.
  $Y = 0.299R + 0.587G + 0.114B$
- the other two vectors:
  $I = 0.596R - 0.275G - 0.321B$ $Q = 0.212R - 0.528G + 0.311B$
- The YIQ **transform**:

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.275 & -0.321 \\ 0.212 & -0.528 & 0.311 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

YIQ Colour Model (Cont.)

NTSC (Analog) Compression Scheme:

- **I** is **red-orange** axis, **Q** is roughly **orthogonal** to **I**.
- Eye is **most sensitive** to **Y**, next to **I**, next to **Q**.

Analog Video Compression Scheme:

- **4 MHz** is allocated to **Y**,
- **1.5 MHz** to **I**,
- **0.6 MHz** to **Q**.

YIQ Colour Space

Original Color Image

Y, I, Q Image Intensities

YUV Color Model

YUV Color Model:

- Digital video standard stablished in 1982
- Video is represented by a sequence of fields (odd and even lines). Two fields make a frame.
- Works in PAL (50 fields/sec) or NTSC (60 fields/sec)
- Uses the **Y**, **U**, **V** colour space

$$Y = 0.299R + 0.587G + 0.114B,$$
$$U = B - Y,$$
$$V = R - Y$$

YUV Color Model (Cont.)

The YUV Transform:

$$\begin{bmatrix} Y \\ U \\ V \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.299 & -0.587 & 0.886 \\ 0.701 & -0.587 & -0.114 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

YCrCb Colour Model

YCrCb (CCIR 601) Colour Model

- **Similar** to **YUV**
- **YUV** is normalised by a scaling

$$Cb = (B - Y)/1.772$$

$$Cr = (R - Y)/1.402$$

$$\begin{bmatrix} Y \\ Cr \\ Cb \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.169 & -0.331 & 0.500 \\ 0.500 & -0.419 & -0.081 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

YCrCb Colour Space

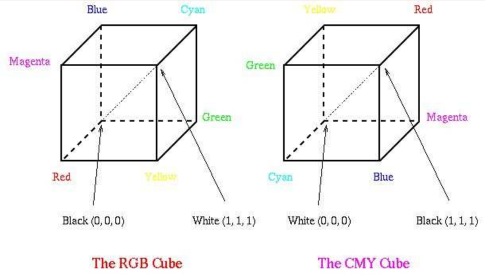
Original Color Image

Y, Cr, Cb Image Intensities

The CMY Colour Model

Printing Colour Models: CMY Colour Model

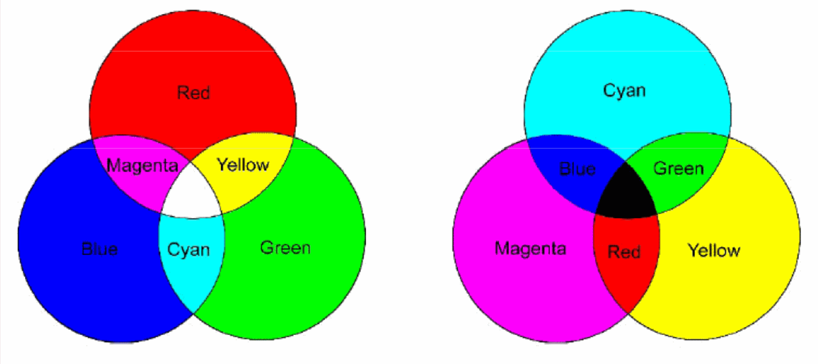
- **Cyan**, **Magenta**, and **Yellow** (**CMY**) are complementary colours of RGB — **Subtractive Primaries**.
- **CMY** model is mostly used in **printing** devices where the colour pigments on the paper absorb certain colours



The CMY Colour Model (cont.)

Additive v. Subtractive Colours

- Combinations of additive and subtractive colours.



Conversion between RGB and CMY

RGB to CMY Conversion:

E.g., convert **White** from (1, 1, 1) in RGB to (0, 0, 0) in CMY.

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} C \\ M \\ Y \end{bmatrix}$$

CMYK Color Model

An improved Printing Colour Model

- Sometimes, an alternative **CMYK** model (**K** stands for **Black**) is used in colour printing
  - *E.g.*, to produce darker black than simply mixing **CMY** (which is never really black!).
- Conversion from CMY to CMYK:

$$
\begin{aligned}
K &= min(C_{CMY}, M_{CMY}, Y_{CMY}), \\
C_{CMYK} &= C_{CMY} - K, \\
M_{CMYK} &= M_{CMY} - K, \\
Y_{CMYK} &= Y_{CMY} - K.
\end{aligned}
$$

CMYK Colour Space

Original Color Image

C, M, Y, K image Intensities

Summary of Colour

- Colour images are encoded as triplets of values.
- Three common systems of encoding in video are RGB, YIQ, and YCrCb.
- Besides the hardware-oriented colour models (i.e., RGB, CMY, YIQ, YUV), HSB (Hue, Saturation, and Brightness, e.g., used in Photoshop) and HLS (Hue, Lightness, and Saturation) are also commonly used.
- YIQ uses properties of the human eye to prioritise information. Y is the black and white (luminance) image, I and Q are the colour (chrominance) images. YUV uses similar idea.
- YUV/YCrCb is a standard for digital video that specifies image size, and decimates the chrominance images (for 4:2:2 video) — **more soon**.

Basics of Video

Types of Colour Video Signals

- **Component video** – each primary is sent as a separate video signal.
    - The primaries can either be **RGB** or a luminance-chrominance transformation of them (e.g., **YIQ**, **YUV**).
    - Best colour reproduction
    - Requires more bandwidth and good synchronisation of the three components
- **Composite video** – colour (chrominance) and luminance signals are mixed into a single carrier wave. Some interference between the two signals is inevitable.
- **S-Video** (Separated video, e.g., in S-VHS) – a compromise between component analog video and the composite video. It uses two lines, one for luminance and another for composite chrominance signal.

NTSC Video

Basic NTSC Video Statistics:

- **525** scan lines per frame, **30** frames per second (or to be exact, 29.97 fps, 33.37 msec/frame)
- Aspect ratio **4:3**
- **Interlaced Video**, each frame is divided into **2 fields**, 262.5 lines/field
- 20 lines reserved for control information at the beginning of each field
  - So a maximum of **485** lines of **visible data**
  - Laser disc and S-VHS had an actual resolution of $\approx$420 lines
  - Ordinary TV: $\approx$320 lines

NTSC Video Colour and Analog Compression

NTSC Analog Video Compression:

- Colour representation:
  - NTSC uses **YIQ** colour model.
  - Composite = $\mathbf{Y} + \mathbf{I}\ \mathit{cos}(\mathbf{Fsc\ t}) + \mathbf{Q}\ \mathit{sin}(\mathbf{Fsc\ t})$, where **Fsc** is the frequency of colour subcarrier
  - Basic Compression Idea
    Eye is most sensitive to Y, next to I, next to Q.
  - This is STILL Analog Compression:
    In NTSC,
    - 4 MHz is allocated to Y,
    - 1.5 MHz to I,
    - 0.6 MHz to Q.
  - Similar (easier to visualise) Compression (Part of) in digital compression — **more soon**

PAL Video

Basic PAL Statistics

- **625** scan lines per frame, **25** frames per second (40 msec/frame)
- Aspect ratio **4:3**
- **Interlaced**, each frame is divided into **2** fields, **312.5** lines/field
- Colour representation:
  - PAL uses **YUV** (**YCrCb**) colour model
  - composite = $\mathbf{Y + 0.492 \times U}\ \mathit{sin}\mathbf{(Fsc\ t) + 0.877 \times V}\ \mathit{cos}\mathbf{(Fsc\ t)}$
- **PAL Analog Video compression**:
  - **5.5 MHz** is allocated to **Y**,
  - **1.8 MHz each** to **U** and **V**.

MATLAB Colour functions

MATLAB's image processing toolbox colour space functions:

- **Colormap manipulation**:
  `colormap` — Set or get colour lookup table
  `rgbplot` —Plot RGB colourmap components
  `cmpermute` — Rearrange colours in colormap.

- **Colour space conversions**:
  `hsv2rgb`/`rgb2hsv` — Convert HSV values/RGB colour space
  `lab2double`/`lab2uint16`/`lab2uint8` — Convert Lab colour values to double etc.
  `ntsc2rgb`/`rgb2ntsc` — Convert NTSC (YIQ)/RGB colour values
  `ycbcr2rgb`/ `rgb2ycbcr` — Convert YCbCr/RGB colour

See forthcoming Lab Class

Chroma Subsampling

Chroma subsampling:

A method that stores an image's (or video frame's) **colour** information at **lower resolution** than its **intensity** information.

- Main Application: **COMPRESSION**
- Used in **JPEG** Image and **MPEG** Video Compression (**More Soon**):
  - **One** (of two) primary **Lossy** sources.

Chroma Subsampling (Cont.)

Exploit traits of **Human visual system (HVS)**

- **HVS** more **sensitive** to variations in **brightness than colour**.
- So devote more bandwidth to **Y than** the color difference components **Cr/I** and **Cb/Q**.
  - **HVS** is less sensitive to the position and motion of color **than luminance**
  - **Bandwidth** can be **optimised** by storing more luminance detail than color detail.
- Reduction results in **almost** no **perceivable** visual difference.

How to Chroma Subsample?

Operate on color difference components

The signal is divided into:

Luma (Y): the **intensity** component and

Chroma: **two color difference** components which we **subsample** in some way to reduce its **bandwidth**

- **Analogous** to **Analog** Video Compression (NTSC or PAL).

How to subsample for chrominance?

The subsampling scheme is commonly expressed as a three part ratio (e.g. **4:2:2**)

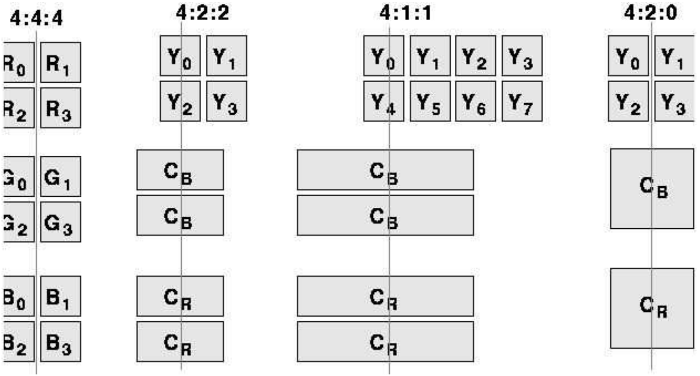
Chroma Subsample 3 Part Ratio Explained

Chroma Subsample 3 Part Ratio:

Each part of the three part ratio is respectively:

1. **Luma (Y) or Red (R)**: **Horizontal** sampling reference (originally, as a multiple of 3.579 MHz in the NTSC analog television system — rounded to 4)
2. **Cr/I/G**: **Horizontal** factor (relative to first digit)
3. **Cb/Q/B**: **Horizontal** factor (relative to first digit), except when **zero**.
   - **Zero** indicates that **Cb (Q/B) horizontal** factor is equal to second digit, **and**,
   - Both **Cr (I/G)** and **Cb (Qb)** are subsampled 2:1 vertically.

Chroma Subsampling Examples

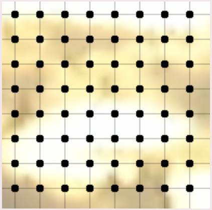


- 4:4:4 — no subsampling in any band — equal ratios.
- 4:2:2 –>Two chroma components are sampled at half the sample rate of luma, horizontal chroma resolution halved.
- 4:1:1 –> Horizontally subsampled by a factor of 4.
- 4:2:0 –> Subsampled by a factor of 2 in both the horizontal and vertical axes

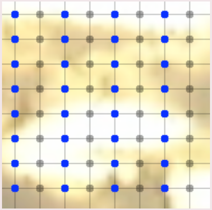
Chroma Subsampling: How to Compute?

Simple Image sub-sampling:

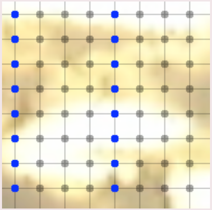
- Simply **different** **frequency sampling** of digitised signal
- Digital Subsampling: For 4:4:4, 4:2:2 and 4:1:1
  Perform 2x2 (or 1x2, or 1x4) chroma subsampling
  - Subsample horizontal and, where applicable, vertical directions
  - *i.e.* Choose every second, fourth pixel value.



4:4:4



4:2:2

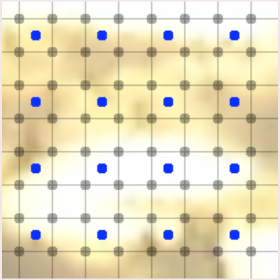


4:1:1

Chroma Subsampling: How to Compute? (Cont.)

4:2:0 Subsampling:

- For **4:2:0**, **Cr** and **Cb** are effectively centred vertically halfway between image rows.:
  - Break the image into **2x2** pixel blocks and
  - Stores the **average** color information for each **2x2** pixel group.



Chroma Subsampling in MATLAB

The MATLAB function `imresize()` readily achieves all our subsampling needs:

```
IMRESIZE Resize image.
   IMRESIZE resizes an image of any type using the specified
   interpolation method. Supported interpolation methods
   include:
        'nearest'  ---  (default) nearest neighbour interpolation
        'bilinear' bilinear interpolation

   B = IMRESIZE(A,M,METHOD) returns an image that is M times the
   size of A. If M is between 0 and 1.0, B is smaller than A. If
   M is greater than 1.0, B is larger than A.
   B = IMRESIZE(A,[MROWS MCOLS],METHOD) returns an image of size
   MROWS-by-MCOLS.
```

After MATLAB colour conversion to **YUV**/**YIQ**, For **U/I** and **V/Q** channels:

- Use `nearest` for **4:2:2** and **4:2:1** and scale the `MCOLS` to half or quarter the size of the image.
- Use `bilinear` (to average) for **4:2:0** and set **scale** to **half**.

See forthcoming Lab worksheet

Digital Chroma Subsampling Errors (1)

This sampling process introduces two kinds of errors:

- A **minor** problem is that colour is typically stored at only half the horizontal and vertical resolution as the original image —*subsampling*.
  This is not a real problem:
  - **Recall**: The human eye has **lower** resolving power for **colour** than for **intensity**.
  - Nearly all digital cameras have lower resolution for colour than for intensity, so there is no high resolution colour information present in digital camera images.

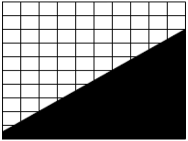
Digital Chroma Subsampling Errors (2)

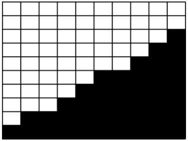
Integer Rounding Errors:

- **Another issue**: The subsampling process demands two conversions of the image:
  - From the original **RGB** representation to an intensity+colour (**YIQ**/**YUV**) representation , and
  - Then back again (**YIQ**/**YUV** –> **RGB**) when the image is displayed.
  - Conversion is done in **integer** arithmetic — some **round-off error** is **introduced**.
    - This is a much smaller effect,
    - But (slightly) affects the colour of (typically) one or two percent of the pixels in an image.
  - Do not compress/recompress videos too often:
    - **Edit the original**

Aliasing in Images

- **Stair-stepping**: Stepped or jagged edges of angled lines, *e.g.*, at the slanted edges of letters.





- **Image Zooming**: changing resolution or not acquiring image in adequate resolution, *e.g.* digital zoom on cameras, digital scanning. (see zoom_alias.m)

Explanation: Simply Application of Nyquist's Sampling Theorem:

Zooming in by a factor n divides the sample resolution by n

Aliasing in Video

Temporal aliasing:

e.g., rotating wagon wheel spokes apparently reversing direction:

Aliasing in Video

Temporal aliasing:

e.g., Optical Illusion - Train Moves Both Ways:

Aliasing in Video: Temporal aliasing (Cont.)

Sampling at Below Nyquist Video Sample Rate

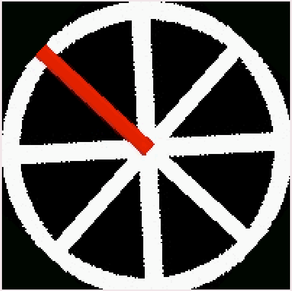
- see aliasing_wheel.m + spokesR.gif:

Below Nyquist Video

Click on image or links to see video.

Aliasing in Video: Temporal aliasing (Cont.)

Sampling at Nyquist Video Sample Rate

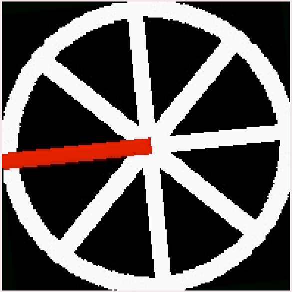


At Nyquist Video

Click on image or links to see video.

Aliasing in Video: Temporal aliasing (Cont.)

Sampling at Above Nyquist Video Sample Rate



Above Nyquist Video

Click on image or links to see video.

Aliasing in Video: Temporal aliasing (Cont.)

Aliasing Explained:

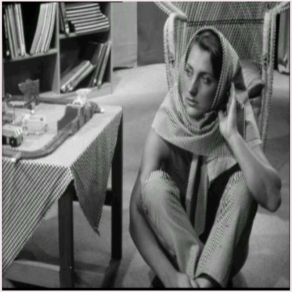
- **'Strobing Effect'**: *e.g.*, rotating wagon wheel spokes apparently reversing direction,
  - See aliasing_wheel.m + spokesR.gif
- The **incorrect** sampling rate "**freezes**" the frames at the wrong moment

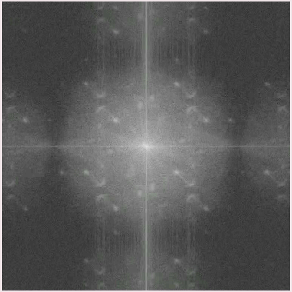
Frame 1 Frame2 Frame3 Frame4 Frame5

Aliasing in Video: Raster scan aliasing

Raster scan aliasing:

e.g., twinkling or strobing effects on sharp horizontal lines, (see raster_aliasing.m + barbara.gif):





Strobing Alias Video

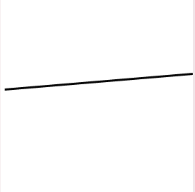
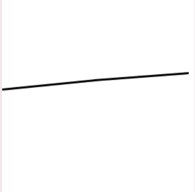
Strobing Alias Frequency Distributions Video

Click on image or links to see video.

Aliasing in Video

Other Video Aliasing Effects

- **Interlacing aliasing**: Some video is interlaced, this effectively halves the sampling frequency. *e.g.*: Interlacing Aliasing effects

- **Image aliasing**: **Stair-stepping**/**Zooming** aliasing effects per frame (as in images).

Explanation: Simply Application of Nyquist's Sampling Theorem