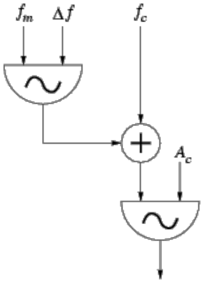
FM (Frequency Modulation) Synthesis

Basic Idea: Timbre of a simple waveform is changed by frequency modulating it with a frequency resulting in a more complex waveform — different-sounding.

Discovered by John Chowning at Stanford University in 1967-68,
Patented in 1975 and was later licensed to Yamaha.
Used in popular 1980s Yamaha Synthesisers: DX7, Casio CZ .....
still in use today



FM (Frequency Modulation) Synthesis (cont.)

- Radio broadcasts use FM in a different way
- FM synthesis is very good at creating both harmonic and inharmonic ('clang', 'twang' or 'bong' noises) sounds
  - For synthesizing harmonic sounds, the modulating signal must have a harmonic relationship to the original carrier signal.
  - As the amount of frequency modulation increases, the sound grows progressively more complex.
  - Through the use of modulators with frequencies that are non-integer multiples of the carrier signal (*i.e.*, non harmonic), bell-like dissonant and percussive sounds can easily be created.

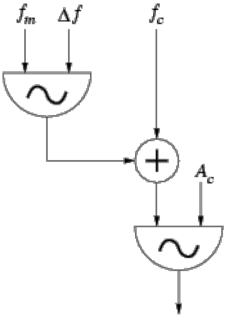
FM (Frequency Modulation) Synthesis (cont.)

- Digital implementation — true analog oscillators difficult to use due to instability
- 1960s origin analog – FM discovered when vibrato sped up to the point that it was creating audible sidebands (perceived as a timbral change) rather than faster warbling (perceived as a frequency change).
- **DX synthesiser FM** - Where both oscillators use Sine waves and are "musically-tuned" frequencies generated from a keyboard

FM Synthesis: Underpinnings

Definitions:

Oscillator:	A device for generating waveforms
Frequency Modulation:	Where the frequency (pitch) of an oscillator (*the Carrier*) is modulated by another oscillator (*the Modulator*)
Carrier Frequency:	The frequency of the oscillator which is being modulated
Modulator Frequency:	The frequency of the oscillator which modulates the Carrier



FM Synthesis: Basic Frequency Modulation

Basic FM Equation:

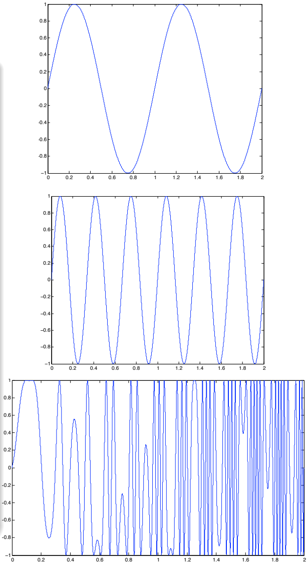
$$e = A \sin(\alpha t + I \sin \beta t)$$

A is the peak amplitude

e is the instantaneous amplitude of the modulated carrier

α and β are the respective carrier and modulator frequencies

I is the modulation index: the ratio of peak deviation to modulator frequency



FM MATLAB Example

MATLAB code to produce basic FM (fm_eg.m), see also fm_eg_plot.m:

fm_eg.m:

```
% Signal parameters
fs = 22050;
T = 1/fs;
dur = 2.0;         % seconds
t = 0:T:dur;       % time vector

% FM parameters
fc = 440;         % center freq
fm = 30;
Imin = 0;  Imax = 20;
I = t.*(Imax - Imin)/dur + Imin;

y = sin(2*pi*fc*t + I.*sin(2*pi*fm*t));
plot(t(1:10000), y(1:10000));

sound(y, fs);
```

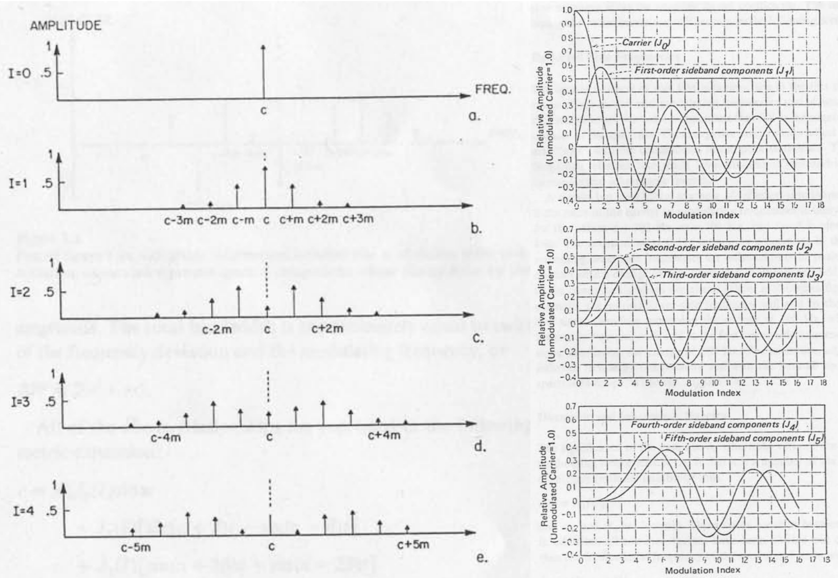
FM Synthesis: Side Frequencies

The **harmonic distribution** of a simple sine wave signal modulated by another sine wave signal can be represented with **Bessel functions**:

$$
\begin{aligned}
e &= A\{J_0 sin\alpha t \\
&+J_1[sin(\alpha+\beta)t - sin(\alpha-\beta)t] \\
&+J_2[sin(\alpha+2\beta)t - sin(\alpha-2\beta)t] \\
&+J_3[sin(\alpha+3\beta)t - sin(\alpha-3\beta)t] \\
&\ldots\}
\end{aligned}
$$

- Provides a basis for a simple mathematical understanding of FM synthesis.
- **Side Frequencies** produced and are related to modulation index, I
  - If $I > 1$ energy is *increasingly stolen* from the carrier but with constant modulation frequency.

FM Synthesis: Side Frequencies (Cont.)



A few insights as to how Bessel functions can help explain why FM synthesis sounds the way it does:

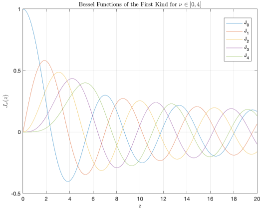
- $J_0(I)$ decides the amplitude of the carrier.
- $J_1(I)$ controls the first upper and lower sidebands.
- Generally, $J_n(I)$ governs the amplitudes of the nth upper and lower sidebands.
- Higher-order Bessel functions start from zero more and more gradually, so higher-order sidebands only have significant energy when I is large.
- The spectral bandwidth increases with I; the upper and lower sidebands grow toward higher and lower frequencies, respectively.
- As I increases, the energy of the sidebands vary much like a damped sinusoid.

MATLAB knows about Bessel functions:

Ch5_3_FM_Synthesis.mlx

```
z = 0:0.1:20;
J = zeros(5,201);
for i = 0:4
    J(i+1,:) = besselj(i,z);
end

plot(z,J)
grid on
legend('J_0','J_1','J_2','J_3','J_4','Location','Best')
title('Bessel Functions of the First Kind for $\nu \in [0, 4]$',
      'interpreter','latex')
xlabel('z','interpreter','latex')
ylabel('$J_\nu(z)$','interpreter','latex')
```



See also: Ch5_3_FM_Synthesis.mlx for further details

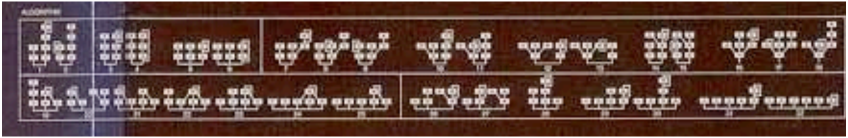
FM Synthesis: Making Complex Sounds

Operators and Algorithms

Operators are just Oscillators in FM Terminology.

- FM synths will have either 4 or 6 Operators.
- **Why so many Operators?**
  Sounds from one Modulator and one Carrier aren't exactly that overwhelmingly complex

Algorithms are the preset combinations of routing available to you.

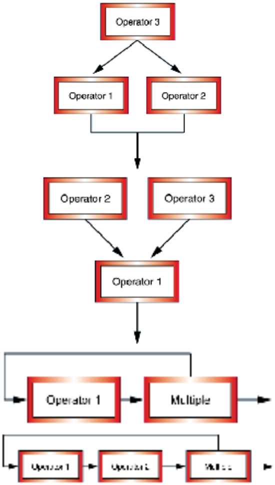


How to connect up Operators?

Multiple Carriers: One oscillator simultaneously modulates two or more carriers

Multiple Modulators: Two or more oscillators simultaneously modulate a single carrier

Feedback: Output of oscillator modulates the same oscillator



Some More FM Waveform Examples

See Ch5_3_FM_Synthesis.mlx for some further practical examples of how to synthesise:

- A sine wave which "compresses" and "uncompress" in time
- A sine wave which undergoes an periodic modulation
- A Bell Sound
- A wood block strike type sound
- Brass sounds