CM2202: Scientific Computing and Multimedia Applications
Laboratory Worksheet (Week 5)

Prof. D. Marshall, Dr. Y. Lai

Aims and Objectives

After working through this worksheet you should be familiar with:

- Vectors operations using MATLAB (both numerical and symbolic).
- Calculus (Differentiation and Integration) using MATLAB.

Work through all the questions below slowly, be careful to assimilate the MATLAB and the ideas behind. They are essential for understanding of much deeper concepts later in the module.

None of the work here is part of the assessed coursework for this module. You may use the remaining time in the lab to start working on the course project.

Vectors

Demo

1. Given two vectors $\mathbf{v} = (1, 2, 5)$ and $\mathbf{w} = (3, -1, 1)$, compute $\mathbf{v} + \mathbf{w}$, $\mathbf{v} - \mathbf{w}$, $3\mathbf{v}$ and $\mathbf{w} \cdot (-1)$.

2. Given two vectors $\mathbf{v} = (3, 2, -1)$ and $\mathbf{w} = (2, -1, 1)$, compute the scalar product $\mathbf{v}.\mathbf{w}$.

3. Given a vector $\mathbf{v} = (3, 4)$, calculate its Euclidean norm.

4. Given two vectors $\mathbf{v} = (1, 2, 3)$ and $\mathbf{w} = (-1, 1, 2)$, compute the cross product $\mathbf{v} \times \mathbf{w}$.

Further Practice

Given two vectors $\mathbf{v} = (-1, 1, 2)$ and $\mathbf{w} = (2, 0, 1)$:

1. Compute $\mathbf{v} + \mathbf{w}$, $\mathbf{v} - \mathbf{w}$, $2\mathbf{v}$ and $-\mathbf{w}$

2. Compute the scalar products $\mathbf{v}.\mathbf{w}$ and $\mathbf{w}.\mathbf{v}$. Explain why they are the same or different.

3. Compute the angle between vectors $\mathbf{v}$ and $\mathbf{w}$.

4. Compute the cross products $\mathbf{v} \times \mathbf{w}$ and $\mathbf{w} \times \mathbf{v}$. Explain why they are the same or different.

5. Let $\mathbf{n}$ be the cross product of $\mathbf{v}$ and $\mathbf{w}$, verify that $\mathbf{n}$ is orthogonal to both $\mathbf{v}$ and $\mathbf{w}$.

6. (*) Use `plot3` to visualise vectors $\mathbf{v}$, $\mathbf{w}$ and $\mathbf{n}$ in the same figure. `plot3` is similar to `plot` but renders in 3D. See `help plot3` for more details (Note: this question is optional.)

The remaining time can be used to build the GUI for the course project. The lab exercise in Week 4 will be useful to provide some basis.

Calculus

Differentiation

1. Using MATLAB `poly()` structures differentiate the following:

   (a) $f(x) = 5x^4$

   (b) $f(x) = 2x^2 + 4x + 1$

2. Using the MATLAB *Symbolic Toolbox* differentiate the following

   (a) $f(x) = 9x + 5$

   (b) $f(f) = 3x^2 - 2x - 1$

   (c) $f(x) = x^3 + x^2 + x$

3. Find the gradient of the following curves at the points indicated.

   (a) $y = x^2 + 3x + 1$ at $(1, 5)$

   (b) $y = \sqrt{(x)}$ at $(4, 2)$

   In each case plot the curve and its tangent at the given point in a MATLAB figure.

4. Find the stationary points for the following functions:

   (a) $f(f) = 3x^2 - 2x - 1$

   (b) $f(x) = x^3 + x^2 + x$

   In each case determine, whether the points are a *maxima* or a *minima* and plot and label them accordingly in a MATLAB figure.

Integration

1. Using MATLAB `poly()` structures integrate the following:

   (a) $f(x) = 5x^4$

   (b) $f(x) = 6x^2 + 4x + 2$

2. Using the MATLAB *Symbolic Toolbox* integrate the following:

   (a) $f(x) = \frac{1}{x^5}$

   (b) $f(f) = (4x + 1)^3$

   (c) $f(x) = \sqrt{x \cos(x)}$

3. Using MATLAB `poly()` structures evaluate the following definite integrals:

   (a) $\int_0^2 x^3 dx$

   (b) $\int_0^3 x^2 + 2x - 1 dx$

4. Using the MATLAB *Symbolic Toolbox* evaluate the following definite integrals:

   (a) $\int_2^4 (x^2 + 4) dx$

   (b) $\int_1^2 \sqrt{x^5} dx$

5. Using the MATLAB, work out the area between the two curves $f(x) = -x^2 + 5x + 15$ and $g(x) = x^2 + 2$

6. Using the MATLAB, work out the area of $f(x) = 3 * x^5 + x^3 - 3$ between $x = -5$ and $x = 5$