A Discussion Game for Grounded Semantics

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Abstract. We introduce an argument-based discussion game where the ability to win the game for a particular argument coincides with the argument being in the grounded extension. Our game differs from previous work in that (i) the number of moves is *linear* (instead of exponential) w.r.t. the strongly admissible set that the game is constructing, (ii) winning the game does not rely on cooperation from the other player (that is, the game is winning strategy based), (iii) a *single* game won by the proponent is sufficient to show grounded membership, and (iv) the game has a number of properties that make it more in line with natural discussion.

1 Introduction

In informal argumentation, discussions play a prominent role. Yet the aspect of discussion has received relatively little attention in formal argumentation theory, especially within the research line of Dung-style argumentation [12]. Whereas other aspects of informal argumentation, like argument schemes [21], claims and conclusions [21, 15], assumptions [2, 13, 30] and preferences [18, 20] have successfully been modelled in the context of (instantiated) Dung-style argumentation, dialectical aspects are often regarded as being part of a research field seperate from inference-based argumentation [22, 24, 25]. The scarce work that does consider dialectical aspects in the context of argument-based entailment tends to do so for the purpose of defining proof procedures [11, 27] that, although useful for software implementation [23], are not meant to actually resemble informal discussion.

One exception to this is the Grounded Persuasion Game of Caminada and Podlaszewski [9], which provides a labelling-based discussion game for grounded semantics. The game is defined such that an argument is in the grounded extension iff there exists at least one game for it that is won by the proponent. However, the Grounded Persuasion Game has a number of shortcomings. For instance, it can be that an argument is in the grounded extension but the proponent does not have a winning strategy for it. That is, although it is possible to win the game, this depends partly on the cooperation of the opponent. Furthermore, in the Grounded Persuasion Game it is the proponent who first introduces the arguments that he later needs to defend against, a phenomenon that rarely occurs in natural discussions other than by mistake.

In the current paper, we present a modified and slightly simplified discussion game for grounded semantics, called the Grounded Discussion Game, that addresses above mentioned shortcomings. Overall, our aim is to provide a discussion game that can be used in the context of human-computer interaction, for the purpose of explaining argument-based inference. This can be helpful to allow users to understand why a particular advice was given by a knowledge-based system, and to examine whether particular objections the user might have can properly be addressed. In this way, we see interactive discussion as an alternative for argument visualisation [28, 29]. Our current work, which is focussed on grounded semantics, fits in a line of research where similar discussion games have been stated also for preferred [7] and stable [10]. With respect to the previously stated games for grounded semantics [27, 3, 19, 9] our aim is to satisfy the following properties:

1. Correctness and completeness for grounded semantics w.r.t. the presence of a winning strategy. It should be the case that an argument is in the grounded extension iff the proponent has a winning strategy for it (unlike for instance [9]).
2. Similarity to natural discussion. No party should be required to introduce arguments that he subsequently has to argue against (unlike for instance [9]). Also, there should be moves in which a player can indicate agreement ("fair enough") at specific points of the discussion (unlike for instance the Standard Grounded Game [27, 3, 19], where such moves are absent).
3. Efficiency. The number of moves should be *linear* in relation to the size of the strongly admissible labelling [5] the game is constructing. This is for instance violated by the Standard Grounded Game [27, 3, 19], where the number of moves can be *exponential* in relation to the size of the strongly admissible labelling the game is constructing (see [5, Section 5.3] for details). A similar observation can be made for other tree-based proof procedures [11, 14].

The remaining part of this paper is structured as follows. First, in Section 2 we provide some preliminaries of argumentation theory. Then, in Section 3 we present our new Grounded Discussion Game, and show that it satisfies the above mentioned properties. We round off in Section 4 with a discussion of the obtained results and how these relate to previous research. Due to space constraints, some of the proofs have been moved to a seperate technical report [6].

2 Formal Preliminaries

Abstract argumentation theory [12] in essence is about how to select nodes from a graph called an *argumentation framework*.

Definition 1 ([12]). *An* argumentation framework *is a pair* (Ar, att) *where* Ar *is a finite set of entities called arguments, and* att *is a binary relation on* Ar. *We say that* A *attacks* B *iff* $(A, B) \in att$.

For current purposes, we apply the labelling-based version of argumentation semantics [8] instead of the original extension-based version of [12]. It should be noticed, however, that an extension is essentially the $\texttt{in}$ labelled part of a labelling [8].

Definition 2 ([8]). *Let (Ar, att) be an argumentation framework. An* argument labelling *is a total function $\mathcal{L}ab : Ar \to \{\texttt{in}, \texttt{out}, \texttt{undec}\}$. An argument labelling is called an* admissible labelling *iff for each $A \in Ar$ it holds that:*

- *if $\mathcal{L}ab(A) = \texttt{in}$ then for each B that attacks A it holds that $\mathcal{L}ab(B) = \texttt{out}$*
- *if $\mathcal{L}ab(A) = \texttt{out}$ then there exists a B that attacks A such that $\mathcal{L}ab(B) = \texttt{in}$*

$\mathcal{L}ab$ is called a complete labelling *iff it is an admissible labelling and for each $A \in Ar$ it also holds that:*

- *if $\mathcal{L}ab(A) = \texttt{undec}$ then there exists a B that attacks A such that $\mathcal{L}ab(B) \neq \texttt{out}$, and there exists no B that attacks A such that $\mathcal{L}ab(B) = \texttt{in}$*

As a labelling is essentially a function, we sometimes write it as a set of pairs. Also, if $\mathcal{L}ab$ is a labelling, we write $\texttt{in}(\mathcal{L}ab)$ for $\{A \in Ar \mid \mathcal{L}ab(A) = \texttt{in}\}$, $\texttt{out}(\mathcal{L}ab)$ for $\{A \in Ar \mid \mathcal{L}ab(A) = \texttt{out}\}$ and $\texttt{undec}(\mathcal{L}ab)$ for $\{A \in Ar \mid \mathcal{L}ab(A) = \texttt{undec}\}$. As a labelling is also a partition of the arguments into sets of $\texttt{in}$-labelled arguments, $\texttt{out}$-labelled arguments and $\texttt{undec}$-labelled arguments, we sometimes write it as a triplet $(\texttt{in}(\mathcal{L}ab), \texttt{out}(\mathcal{L}ab), \texttt{undec}(\mathcal{L}ab))$.

Definition 3 ([8]). *Let $\mathcal{L}ab$ be a complete labelling of argumentation framework $AF = (Ar, att)$. $\mathcal{L}ab$ is said to be the grounded labelling iff $\texttt{in}(\mathcal{L}ab)$ is minimal (w.r.t. set inclusion) among all complete labellings of AF.*

The discussion game to be presented in Section 3 of this paper is based on the concept of strong admissibility [1, 5]. Hence, we briefly recall its basic definitions.

Definition 4 ([5]). *Let $\mathcal{L}ab$ be an admissible labelling of argumentation framework (Ar, att). A* min-max numbering *is a total function $\mathcal{MM}_{\mathcal{L}ab} : \texttt{in}(\mathcal{L}ab) \cup \texttt{out}(\mathcal{L}ab) \to \mathbb{N} \cup \{\infty\}$ such that for each $A \in \texttt{in}(\mathcal{L}ab) \cup \texttt{out}(\mathcal{L}ab)$ it holds that:*

- *if $\mathcal{L}ab(A) = \texttt{in}$ then $\mathcal{MM}_{\mathcal{L}ab}(A) = max\ (\{\mathcal{MM}_{\mathcal{L}ab}(B) \mid B$ attacks A and $\mathcal{L}ab(B) = \texttt{out}\}) + 1$ (with $max(\emptyset)$ defined as 0)*
- *if $\mathcal{L}ab(A) = \texttt{out}$ then $\mathcal{MM}_{\mathcal{L}ab}(A) = min\ (\{\mathcal{MM}_{\mathcal{L}ab}(B) \mid B$ attacks A and $\mathcal{L}ab(B) = \texttt{in}\}) + 1$ (with $min(\emptyset)$ defined as ∞)*

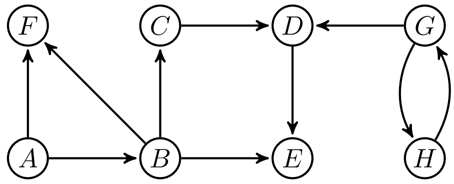
Theorem 1 ([5]). *Every admissible labelling has a unique min-max numbering.*

Definition 5 ([5]). *A* strongly admissible labelling *is an admissible labelling whose min-max numbering yields natural numbers only (so no argument is numbered ∞).*

Theorem 2 ([5]). *An argument is labelled $\texttt{in}$ (resp. $\texttt{out}$) by at least one strongly admissible labelling iff it is labelled $\texttt{in}$ (resp. $\texttt{out}$) by the grounded labelling.*

As an example, consider the argumentation framework shown below, which we refer to as AF_{ex}. Here $\mathcal{L}ab_1 = (\{A, C, E, G\}, \{B, D, H\}, \{F\})$ is an admissible (though not complete) labelling with associated min-max numbering $\mathcal{MM}_{\mathcal{L}ab_1} = \{(A\colon 1), (B\colon 2), (C\colon 3), (D\colon 4), (E\colon 5), (G\colon \infty), (H\colon \infty)\}$, which implies that $\mathcal{L}ab_1$ is not strongly admissible. Furthermore, $\mathcal{L}ab_2 = (\{A, C, E\}, \{B, D, F\}, \{G, H\})$ is an admissible (and complete) labelling with associated min-max numbering $\mathcal{MM}_{\mathcal{L}ab_2} = \{(A\colon 1), (B\colon 2), (C\colon 3), (D\colon 4), (E\colon 5), (F\colon 2)\}$, which implies that $\mathcal{L}ab_2$ is indeed a strongly admissible labelling.

From Theorem 2, together with the fact that the grounded extension consists of the `in`-labelled arguments of the grounded labelling [8], it follows that to show that an argument is in the grounded extension, it is sufficient to construct a strongly admissible labelling that labels the argument `in`.



3 The Grounded Discussion Game

The Grounded Discussion Game that we will define in the current section has two players (proponent and opponent) and is based on four different moves, each of which has an argument as a parameter.

HTB(A) ("A has to be the case")
: With this move, the proponent claims that A has to be labelled `in` by every complete labelling, and hence also has to be labelled `in` by the grounded labelling.

CB(B) ("B can be the case, or at least cannot be ruled out")
: With this move, the opponent claims that B does not have to be labelled `out` by every complete labelling. That is, the opponent claims there exists a complete labelling where B is labelled `in` or `undec`, and that B is therefore not labelled `out` by the grounded labelling.

CONCEDE(A) ("I agree that A has to be the case")
: With this move, the opponent indicates that he now agrees with the proponent (who previously did a *HTB*(A) move) that A has to be the case (labelled `in` by every complete labelling, including the grounded).

RETRACT(B) ("I give up that B can be the case")
: With this move, the opponent indicates that he no longer believes that B can be `in` or `undec`. That is, the opponent acknowledges that B has to be labelled `out` by every complete labelling, including the grounded.

One of the key ideas of the discussion game is that the proponent has burden of proof. He has to establish the acceptance of the main argument and make sure the discussion does not go around in circles. The opponent merely has to cast sufficient doubts.

The game starts with the proponent uttering a HTB statement. After each HTB statement (either the first one or a subsequent one) the opponent utters a sequence of one or more CB, $CONCEDE$ and $RETRACT$ statements, after which the proponent again utters an HTB statement, etc. In AF_{ex} the discussion could go as follows.

(1) P: $HTB(C)$	(4) O: $CONCEDE(A)$
(2) O: $CB(B)$	(5) O: $RETRACT(B)$
(3) P: $HTB(A)$	(6) O: $CONCEDE(C)$

In the above discussion, C is called *the main argument* (the argument the discussion starts with). The discussion above ends with the main argument being conceded by the opponent, so we say that the proponent wins the discussion.

As an example of a discussion that is lost by the proponent, it can be illustrative to examine what happens if, still in AF_{ex}, the proponent claims that B has to be the case.

(1) P: $HTB(B)$ (2) O: $CB(A)$

After the second move, the discussion is terminated, as the proponent cannot make any further move, since A does not have any attackers. This brings us to the precise preconditions of the discussion moves.

$HTB(A)$ Either this is the first move, or the previous move was $CB(B)$, where A attacks B, and no $CONCEDE$ or $RETRACT$ move is applicable.

$CB(A)$ A is an attacker of the last $HTB(B)$ statement that is not yet conceded, the directly preceeding move was not a CB statement, argument A has not yet been retracted, and no $CONCEDE$ or $RETRACT$ move is applicable.

$CONCEDE(A)$ There has been a $HTB(A)$ statement in the past, of which every attacker has been retracted, and $CONCEDE(A)$ has not yet been moved.

$RETRACT(A)$ There has been a $CB(A)$ statement in the past, of which there exists an attacker that has been conceded, and $RETRACT(A)$ has not yet been moved.

Apart from the preconditions mentioned above, all four statements also have the additional precondition that no HTB-CB repeats have occurred. That is, there should be no argument for which HTB has been uttered more than once, CB has been uttered more than once, or both HTB and CB have been uttered. In the first and second case, the discussion is going around in circles, which the proponent has to prevent as he has burden of proof. In the third case, the proponent has been contradicting himself, as his statements are not conflict-free. In each of these three cases, the discussion comes to an end with no move being applicable anymore. The above conditions are made formal as follows.

Definition 6. *Let $AF = (Ar, att)$ be an argumentation framework. A* grounded discussion *is a sequence of discussion moves constructed by applying the following principles.*

BASIS (*HTB*) *If $A \in Ar$ then $[HTB(A)]$ is a grounded discussion.*

STEP (*HTB*) *If $[M_1, \ldots, M_n]$ ($n \geq 1$) is a grounded discussion without HTB-CB repeats,*[1] *and no CONCEDE or RETRACT move is applicable,*[2] *and $M_n = CB(A)$ and B is an attacker of A then $[M_1, \ldots, M_n, HTB(B)]$ is also a grounded discussion.*

STEP (*CB*) *If $[M_1, \ldots, M_n]$ ($n \geq 1$) is a grounded discussion without HTB-CB repeats, and no CONCEDE or RETRACT move is applicable, and M_n is not a CB move, and there is a move $M_i = HTB(A)$ ($i \in \{1 \ldots n\}$) such that the discussion does not contain $CONCEDE(A)$, and for each move $M_j = HTB(A')$ ($j > i$) the discussion contains a move $CONCEDE(A')$, and B is an attacker of A such that the discussion does not contain a move $RETRACT(B)$, then $[M_1, \ldots, M_n, CB(B)]$ is a grounded discussion.*

STEP (*CONCEDE*) *If $[M_1, \ldots, M_n]$ ($n \geq 1$) is a grounded discussion without HTB-CB repeats, and $CONCEDE(B)$ is applicable then $[M_1, \ldots, M_n, CONCEDE(B)]$ is a grounded discussion.*

STEP (*RETRACT*) *If $[M_1, \ldots, M_n]$ ($n \geq 1$) is a grounded discussion without HTB-CB repeats, and $RETRACT(B)$ is applicable then $[M_1, \ldots, M_n, RETRACT(B)]$ is a grounded discussion.*

It can be observed that the preconditions of the moves are such that a proponent move (*HTB*) can never be applicable at the same moment as an opponent move (*CB*, *CONCEDE* or *RETRACT*). That is, proponent and opponent essentially take turns in which each proponent turn consists of a single *HTB* statement, and every opponent turn consists of a sequence of *CONCEDE*, *RETRACT* and *CB* moves.

Definition 7. *A grounded discussion $[M_1, \ldots, M_n]$ is called* terminated *iff there exists no move M_{n+1} such that $[M_1, \ldots, M_n, M_{n+1}]$ is a grounded discussion. A terminated grounded discussion (with A being the main argument) is won by the proponent iff the discussion contains $CONCEDE(A)$, otherwise it is won by the opponent.*

To illustrate why the discussion has to be terminated after the occurrence of a *HTB-CB* repeat, consider the following discussion in AF_{ex}.

(1) P: $HTB(G)$
(2) O: $CB(H)$
(3) P: $HTB(G)$

After the third move, an *HTB-CB* repeat occurs and the discussion is terminated (opponent wins). Hence, termination after a *HTB-CB* repeat is necessary to prevent the discussion from going on perpetually.

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[1] We say that there is a *HTB-CB* repeat iff $\exists i, j \in \{1 \ldots n\} \exists A \in Ar : (M_i = HTB(A) \vee M_i = CB(A)) \wedge (M_j = HTB(A) \vee M_j = CB(A)) \wedge i \neq j$.

[2] A move $CONCEDE(B)$ is applicable iff the discussion contains a move $HTB(A)$ and for every attacker A of B the discussion contains a move $RETRACT(B)$, and the discussion does not already contain a move $CONCEDE(B)$. A move $RETRACT(B)$ is applicable iff the discussion contains a move $CB(B)$ and there is an attacker A of B such that the discussion contains a move $CONCEDE(A)$, and the discussion does not already contain a move $RETRACT(B)$.

Theorem 3. *Every discussion will terminate after a finite number of steps.*

From the fact that a discussion terminates after an *HTB-CB* repeat, the following result follows.

Lemma 1. *No discussion can contain a CONCEDE and RETRACT move for the same argument.*

3.1 Soundness

Now that the workings of the game have been outlined, the next step will be to formally prove its soundness and completeness w.r.t. grounded semantics. We start with soundness: if a discussion is won by the proponent, then the main argument is in the grounded extension. In order to prove this, we first have to introduce the notions of the proponent labelling and the opponent labelling.

Definition 8. *Let* $[M_1, \ldots, M_n]$ *be a grounded discussion (in argumentation framework* (Ar, att)*) without any HTB-CB repeats.*
The proponent labelling $\mathcal{L}ab_P$ *is defined as*
$\mathtt{in}(\mathcal{L}ab_P) = \{A \mid \exists i \in \{1 \ldots n\}\colon M_i = HTB(A)\}$
$\mathtt{out}(\mathcal{L}ab_P) = \{A \mid \exists i \in \{1 \ldots n\}\colon M_i = CB(A)\}$
$\mathtt{undec}(\mathcal{L}ab_P) = Ar \setminus (\mathtt{in}(\mathcal{L}ab_P) \cup \mathtt{out}(\mathcal{L}ab_P))$
The opponent labelling $\mathcal{L}ab_O$ *is defined as*
$\mathtt{in}(\mathcal{L}ab_O) = \{A \mid \exists i \in \{1 \ldots n\}\colon M_i = CONCEDE(A)\}$
$\mathtt{out}(\mathcal{L}ab_O) = \{A \mid \exists i \in \{1 \ldots n\}\colon M_i = RETRACT(A)\}$
$\mathtt{undec}(\mathcal{L}ab_O) = Ar \setminus (\mathtt{in}(\mathcal{L}ab_O) \cup \mathtt{out}(\mathcal{L}ab_O))$

Notice that the well-definedness of $\mathcal{L}ab_O$ in Definition 8 does not depend on the absence of *HTB-CB* repeats (this is due to Lemma 1) whereas the well-definedness of $\mathcal{L}ab_P$ does. When applying $\mathcal{L}ab_O$, we will therefore often do so without having ruled out any *HTB-CB* repeats, as for instance in the following theorem.

Theorem 4. *Let* $\mathcal{L}ab_O$ *be the opponent's labelling w.r.t. discussion* $[M1, \ldots, M_n]$*.* $\mathcal{L}ab_O$ *is strongly admissible.*

Theorem 4 states that at any stage of the discussion, $\mathcal{L}ab_O$ is strongly admissible (this can be proved by induction over the number of *CONCEDE* and *RETRACT* moves [6]). Hence, when the game is terminated and won by the proponent, we have a strongly admissible labelling where (by definition of winning) the main argument is labelled `in`. It then follows (Theorem 2) that the main argument is labelled `in` by the grounded labelling and is therefore an element of the grounded extension [8], leading to the following theorem.

Theorem 5. *Let* $[M_1, \ldots, M_n]$ *be a terminated grounded discussion, won by the proponent, with main argument A. It holds that A is in the grounded extension.*

As an aside, although it is possible to infer that an argument is in the grounded extension when the proponent wins a discussion (Theorem 5) we cannot infer that an argument is *not* in the grounded extension when the proponent loses a discussion. This is because loss of a game could be due to the proponent following a flawed strategy. For instance, in AF_{ex} one could have the following discussion:

(1) P: $HTB(E)$	(4) O: $CB(H)$
(2) O: $CB(D)$	(5) P: $HTB(G)$
(3) P: $HTB(G)$	

The discussion is terminated at step (5) due to a *HTB*-*CB* repeat ($HTB(G)$). The main argument is not conceded, so the proponent loses. Still the proponent could have won by moving $HTB(C)$ instead of $HTB(G)$ at step (3).

3.2 Completeness

Now that the soundness of the game has been shown, we shift our attention to completeness. The obvious thing to prove regarding completeness would be the converse of Theorem 5: if A is in the grounded extension, then there exists a discussion won by the proponent with A as the main argument. However, our aim is to prove a slightly stronger property. Instead of there being just a single discussion won by the proponent, which might be due to the opponent actually providing cooperation during the game, we require the proponent to have a winning strategy. That is, when an argument is in the grounded extension, the proponent will be able to win the game, irrespective of how the opponent choses to play it.

The idea is that a strongly admissible labelling (for instance the grounded labelling) with its associated min-max numbering can serve as a roadmap for winning the discussion. The proponent will be able to win if, whenever he has to do a *HTB* move, he prefers to use an `in` argument with the lowest min-max number that attacks the directly preceding *CB* move. We refer to this as a *lowest number strategy.*[3]

We first observe that when applying such a strategy, the game stays within the boundaries of the strongly admissible labelling (that is, within its `in` and `out` labelled part). As long as each *HTB* move of the proponent is related to an `in`-labelled argument, it follows that all the attackers are labelled `out` (Definition 2, first bullet) so each *CB* move the opponent utters in response will be related to an `out`-labelled argument. This `out`-labelled argument will then have at least one `in`-labelled attacker (Definition 2, second bullet) as a candidate for the proponent's subsequent *HTB* move.

The next thing to be observed is that when the proponent applies a lowest number strategy, the game will not terminate due to any *HTB*-*CB* repeats. This

[3] We write "*a* lowest number strategy" instead of "*the* lowest number strategy" as a lowest number strategy might not be unique due to different lowest numbered `in`-labelled arguments being applicable at a specific point. In that case it suffices to pick an arbitrary one.

is due to the facts that (1) after a move $HTB(A)$ is played (for some argument A) all subsequent CB and HTB moves will be related to arguments with lower min-max numbers than A until a move $CONCEDE(A)$ is played, and (2) after a move $CB(A)$ is played (for some argument A), all subsequent HTB and CB moves will be related to arguments with lower min-max numbers than A, until a move $RETRACT(A)$ is played. We refer to [6] for details.

Lemma 2. *If the proponent uses a lowest number strategy, then no HTB-CB repeats occur.*

We are now ready to present the main result regarding completeness of the discussion game.

Theorem 6. *Let A be an argument in the grounded extension of argumentation framework* (Ar, att). *If the proponent uses a lowest number strategy, he will win the discussion for main argument A.*

Theorem 6 partly follows from the facts that each discussion will terminate in a finite number of moves (Theorem 3) and, as the proponent uses a lowest number strategy, termination cannot be due to any *HTB*-*CB* repeat (Lemma 2). We refer to [6] for details. As the presence of a winning strategy trivially implies the presence of at least one discussion that is won by the proponent, we immediately obtain the following result.

Corollary 1. *Let A be an argument in the grounded extension of argumentation framework* (Ar, att). *There exists at least one terminated grounded discussion, won by the proponent, for main argument A.*

3.3 Efficiency

Now that soundness and completeness of the game have been shown, we proceed to examine its efficiency. Theorem 3 states that every discussion will terminate, and we are interested in how many steps are required for this. For this, we need the following lemma (proof in [6]).

Lemma 3. *Let A be an argument in the grounded extension of argumentation framework* (Ar, att). *When the proponent uses a lowest number strategy for the discussion of A, then once the game is terminated it holds that* $\mathcal{L}ab_O = \mathcal{L}ab_P$.

In a terminated discussion yielded by a lowest number strategy, there exists a one-to-one relation between HTB moves and arguments in $\mathtt{in}(\mathcal{L}ab_P)$ (as no HTB move is repeated, as there are no *HTB*-*CB* repeats), a one-to-one relationship between CB moves and arguments in $\mathtt{out}(\mathcal{L}ab_P)$ (as no CB move is repeated, as there are non *HTB*-*CB* repeats), a one-to-one relation between $CONCEDE$ moves and arguments in $\mathtt{in}(\mathcal{L}ab_O)$ (as no $CONCEDE$ move can be repeated) and a one-to-one relation between $RETRACT$ moves and arguments in $\mathtt{out}(\mathcal{L}ab_O)$ (as no $RETRACT$ move can be repeated). Hence, the total number of moves is $|\mathtt{in}(\mathcal{L}ab_P)| + |\mathtt{out}(\mathcal{L}ab_P)| + |\mathtt{in}(\mathcal{L}ab_O)| + |\mathtt{out}(\mathcal{L}ab_O)|$. Due to the facts that

$\texttt{in}(\mathcal{L}ab_P) \cap \texttt{out}(\mathcal{L}ab_P) = \emptyset$, $\texttt{in}(\mathcal{L}ab_O) \cap \texttt{out}(\mathcal{L}ab_O) = \emptyset$, and $\mathcal{L}ab_P = \mathcal{L}ab_O$ (Lemma 3), this is equivalent to $2 \cdot |\texttt{in}(\mathcal{L}ab_P) \cup \texttt{out}(\mathcal{L}ab_P)|$ and to $2 \cdot |\texttt{in}(\mathcal{L}ab_O) \cup \texttt{out}(\mathcal{L}ab_O)|$, so to two times the *size* [5] of either $\mathcal{L}ab_P$ or $\mathcal{L}ab_O$.

Theorem 7. *Let A be an argument in the grounded extension of argumentation framework $AF = (Ar, att)$. When the proponent uses a lowest number strategy for A, the resulting terminated discussion will have a number of moves that is linear w.r.t. the size of the strongly admissible labelling that is has been constructed.*

4 Discussion and Related Work

As was shown in Section 3, the Grounded Discussion Game is based on the concept of strong admissibility. In essence, it constructs a strongly admissible labelling where the main argument is labelled `in` (Theorem 4). Moreover, the presence of a strongly admissible labelling provides the proponent with a winning strategy for the game (Theorem 6). These observations make it possible to compare the Grounded Discussion Game with two previously defined games that are also based on strong admissibility: the Standard Grounded Game [27, 3, 19] and the Grounded Persuasion Game [9].

4.1 The Standard Grounded Game

The Standard Grounded Game (SGG) [27, 3, 19] is one of the earliest dialectical proof procedures for grounded semantics. Each game[4] consists of a sequence $[A_1, \dots, A_n]$ ($n \geq 1$) of arguments, moved by the proponent and opponent taking turns, with the proponent starting. That is, a move A_i ($i \in \{1 \dots n\}$) is a proponent move iff i is odd, and an opponent move iff i is even. Each move, except the first one, is an attacker of the previous move. In order to ensure termination even in the presence of cycles, the proponent is not allowed to repeat any of his moves. A game is terminated iff no next move is possible; the player making the last move wins.

As an example, in AF_{ex} $[C, B, A]$ is terminated and won by the proponent (as A has no attackers, the opponent cannot move anymore) whereas $[G, H]$ is terminated and won by the opponent (as the only attacker of H is G, which the proponent is not allowed to repeat). It is sometimes possible for the proponent to win a game even if the main argument is not in the grounded extension. An example would be $[F, B, A]$. This illustrates that in order to show that an argument is in the grounded extension, a single game won by the proponent is not sufficient. Instead, what is needed is a *winning strategy.* This is essentially a tree in which each node is associated with an argument such that (1) each path from the root to a leaf constitutes a terminated discussion won by the proponent, (2) the children of each proponent node (a node corresponding with a proponent move) coincide with all attackers of the associated argument, and

[4] What we call an SGG game is called a "line of dispute" in [19].

(3) each opponent node (a node corresponding with an opponent move) has precisely one child, whose argument attacks the argument of the opponent node.

It has been proved that an argument is in the grounded extension iff the proponent has a winning strategy for it in the SGG [27, 3]. Moreover, it has also been shown that an SGG winning strategy defines a strongly admissible labelling, when labelling each argument of a proponent node `in`, each argument of an opponent node `out` and all remaining arguments `undec` [5].

As an example, in AF_{ex} the winning strategy for argment E would be the tree consisting of the two branches $E-B-A$ and $E-D-C-B-A$, thus proving its membership of the grounded extension by yielding the strongly admissible labelling $(\{A, C, E\}, \{B, D\}, \{F, G, H\})$. As can be observed from this example, a winning strategy of the SGG can contain some redundancy when it comes to multiple occurrences of the same arguments in different branches. In the current example, the redundancy is relatively mild (consisting of just the two arguments A and B) but other cases have been found where the SGG requires a number of moves in the winning strategy that is *exponential* w.r.t. the size of the strongly admissible labelling the winning strategy is defining [5, Figure 2].[5] Hence, one of the advantages of our newly defined GDG compared to the SGG is that we go from an exponential [5, Figure 2] to a linear (Theorem 7) number of moves.[6]

4.2 The Grounded Persuasion Game

One of the main aims of the Grounded Persuasion Game (GPG) [9] was to bring the proof procedures of grounded semantics more in line with Mackenzie-style dialogue theory [16, 17] The game has two participants (P and O) and four types of moves: `claim` (the first move in the discussion, with which P utters the main claim that a particular argument has to be labelled `in`), `why` (with which O asks why a particular argument has to be labelled in a particular way), `because` (with which P explains why a particular argument has to be labelled a particular way) and `concede` (with which O indicates agreement with a particular statement of P). During the game, both P and O keep *commitment stores*, partial labellings (which we will refer to as $\mathcal{P}$ and $\mathcal{O}$) which keep track of which arguments they think are `in` and `out` during the course of the discussion. For P, a commitment is added every time he utters a `claim` or `because` statement. For O, a commitment is added every time he utters a `concede` statement. An *open issue* is an argument where only one player has a commitment. Some of the key rules of the Grounded Persuasion Game are as follows (full details in [9]).

[5] A similar observation can be made for other tree-based proof procedures [11, 14].

[6] As each move contains a single argument, this means the "communication complexity" (the total number of arguments that needs to be communicated) is also linear. This contrasts with the computational complexity of playing the game, which is polynomial ($O(n^3)$, where n is the number of arguments) due to the fact that selecting the next move can have $O(n^2)$ complexity (see [6] for details). This is still less than when applying Standard Grounded Game, whose overall complexity would be exponential (even if each move could be selected in just one step) due to the requirement of a winning strategy, which as we have seen can be exponential in size.

- If O utters a `why` `in`(A) statement (resp. a `why` `out`(A) statement) then P has to reply with `because` `out`($B_1, \ldots, B_n$) where $B_1, \ldots, B_n$ are all attackers of A (resp. with `because` `in`(B) where B is an attacker of A).
- Any `why` statement of O has to be related to the most recently created open issue in the discussion.
- A `because` statement is not allowed to use an argument that is already an open issue.
- Once O has enough evidence to agree with P that a particular argument has to be labelled `in` (because for each of its attackers, O is already committed that the attacker is labelled `out`) or has to be labelled `out` (because it has an attacker of which O is already committed that it is labelled `in`), O has to utter the relevant `concede` statement immediately.

Unlike the SGG, in the GPG it is not necessary to construct a winning strategy to show grounded membership. Instead, an argument A is in the grounded extension iff there exists *at least one game* that starts with P uttering "`claim` `in`(A)" and is won by P [9].[7]

As a general property of the Grounded Persuasion Game, it can be observed that at every stage of the discussion, O's commitment store $\mathcal{O}$ is an admissible labelling [9].[8]

As an example, for argument E in AF_{ex} the discussion could go as follows.

	`in`($\mathcal{P}$)	`out`($\mathcal{P}$)	`in`($\mathcal{O}$)	`out`($\mathcal{O}$)
(1) P: claim `in`(E)	E			
(2) O: why `in`(E)	E			
(3) P: because `out`(B, D)	E	B,D		
(4) O: why `out`(B)	E	B,D		
(5) P: because `in`(A)	E,A	B,D		
(6) O: concede `in`(A)	E,A	B,D	A	
(7) O: concede `out`(B)	E,A	B,D	A	B
(8) O: why `out`(D)	E,A	B,D	A	B
(9) P: because `in`(C)	E,A,C	B,D	A	B
(10)O: concede `in`(C)	E,A,C	B,D	A,C	B
(11)O: concede `out`(D)	E,A,C	B,D	A,C	B,D
(12)O: concede `in`(E)	E,A,C	B,D	A,C,E	B,D

In the above game, the main claim `in`(E) is conceded so the proponent wins. As was mentioned above, a "because" statement is not allowed to use an argument that is already an open issue. This is to ensure termination even in the presence of cycles. However, this condition has an undesirable side effect. Consider what happens when, at move (4) of the above discussion, the opponent would have decided to utter "why `out`(D)" instead of "why `out`(B)".

[7] A discussion is won by P iff at the end of the game O is committed that the argument the discussion started with is labelled `in`.

[8] That is, if one regards all arguments where O does not have any commitments to be labelled `undec`.

(4′) O: why $\texttt{out}(D)$	E	B,D
(5′) P: because $\texttt{in}(C)$	E,C	B,D
(6′) O: why $\texttt{in}(C)$	E,C	B,D

After move (6′) the proponent cannot reply with "because $\texttt{out}(B)$" as $\texttt{out}(B)$ is an open issue, so the game is terminated (according to the rules of [9]) without the main claim being conceded, meaning the proponent loses. Moreover, there is nothing the proponent could have done differently in order to win the game, in spite of E being in the grounded extension. One of the advantages of our currently defined Grounded Discussion Game is that such anomalies cannot occur (Theorem 6). Once the proponent utters $HTB(E)$ he can win the game, regardless of whether the opponent responds with $CB(B)$ or with $CB(D)$.

Another difference between the GPG and our currently defined GDG is related to the player who introduces the counterarguments in the discussion. In the GPG this is always the proponent, who for instance explicitly has to list all the attackers against an argument he is actually trying to defend (like "P: because $\texttt{out}(B, A)$" in the above discussion). However, in natural discussion it would be rare for any participant to provide counterarguments against his own position, other than by mistake. The GDG, however, is such that in a game won by the proponent, each of the counterarguments uttered against proponent's position is uttered by the opponent.

4.3 Summary and Analysis

Overall, the differences between our approach and the other games are summarised in the following table.

	SGG	GPG	GDG
number of moves needed to show strong admissibility	exp [5]	linear [5]	linear (Th. 7)
supports RETRACT and/or CONCEDE moves	no	yes	yes
both proponent and opponent introduce arguments	yes	no	yes
single successful game implies grounded membership	no	yes	yes
grounded membership implies ∃ winning strategy	yes	no	yes

Apart from the technical considerations mentioned above, the research agenda of developing argument-based discussion games is also relevant because it touches some of the foundations of argumentation theory. Whereas for instance classical logic entailment is based on the notion of *truth*, this notion simply does not exist in abstract argumentation and would be problematic even in instantiated argumentation.[9] But if not truth, then what actually is it that is actually yielded

[9] For instance, if a conclusion is considered justified in ASPIC+ [26, 21], does this imply the conclusion is also *true*?

by formal argumentation theory? Our view is that argumentation theory yields what can be defended in rational discussion. As our Grounded Discussion Game is essentially a form of persuasion dialogue [31] we have shown that grounded semantics can be seen as a form of persuasion dialogue. Furthermore, Caminada et al. have for instance shown that (credulous) preferred semantics can be seen as a particular form of Socratic dialogue [4, 7]. Hence, different argumentation semantics correspond to different types of discussion [7], an observation that is not just relevant for philosophical reasons, but also opens up opportunities for argument-based human computer interaction. In further research we hope to report on whether engaging in the Grounded Discussion Game increases people's trust in particular forms of argument-based inference. An implementation, that can serve as the basis for this, is currently under development.

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