Breeding Permutations for Minimum Span Frequency Assignment

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Abstract

This paper describes a genetic algorithm for solving the minimum span frequency assignment problem (MSFAP). The MSFAP involves assigning frequencies to each transmitter in a region, subject to a number of constraints being satisfied, such that the span, i.e. range of frequencies used, is minimised. The technique involves finding an ordering of the transmitters for use in a sequential (greedy) assignment process. Results are given for several practical problem instances.

1 Introduction

The management of the radio spectrum would be eased if frequencies could always be assigned for a particular purpose in an optimum or near-optimum manner. However, the assignment of radio frequencies for problems of a practical size remains a considerable challenge.

The primary objective in minimum span frequency assignment is to assign radio frequencies to a number of transmitters subject to a number of constraints, such that no interference is suffered, and the range of frequencies used, i.e. the difference between the largest and smallest frequency used, is minimised (this is the *span* of the assignment). The general assignment problem is classified computationally as *NP*-hard. Hence, there is no known algorithm that can generate a guaranteed optimal solution in an execution time that may be expressed as a finite polynomial of the problem dimension.

Computing methods based on exact algorithms and graph theory are successful for small problems but out of the question for large problems. Methods based on the so-called sequential heuristics, which mimic the way the problem might be solved manually, are fast enough for large problems but give results which are well short of the best possible.

The purpose of this paper is to explore the possibility of combining a genetic algorithm with sequential assignment methods. Traditionally, metaheuristics have been applied to an initial solution consisting of an assignment of frequencies to transmitters and then attempting to minimise the number of constraint violations [2, 4, 8]. Here, the iterative transformations are applied to permutations of transmitters. A simple sequential assignment algorithm is then applied to each of these permutations to produce an allocation of frequencies that does not violate any constraints.

1.1 Interference and Constraints

In order to model interference, constraints are imposed on the assignment. Pairs of transmitters can interfere with each other when the assigned frequencies are the same or close together. This can happen when transmitters are at the same location or within a few tens of metres of each other *(co-site interference)*, or when equipment is at a distance of several kilometres or more *(far-site interference)*.

The constraints are due to the following interference mechanisms and can be expressed in terms of equalities or inequalities involving no more than four frequencies.

Co-channel and Harmonic constraints: This is the most important factor in the consideration of far-site interference. A pair of transmitters located at different sites must not be assigned the same frequency, or harmonics of each other, unless they are sufficiently geographically separated. This gives rise to constraints of the form:

$$f_i \neq nf_j \quad \text{for} \quad n = 1, 2, 3, ...$$

When $n = 1$ we have a co-channel constraint.

Adjacent channel constraints: When a transmitter and a receiver are tuned to similar frequencies (normally within three channels of each other),

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there is still the potential for interference. Therefore a number of constraints arise of the following form:

$$|f_i - f_j| > m$$

for some value of m, where m is the number of channels separation.

Co-site frequency separation: Any pair of frequencies at a site must be separated by a certain fixed amount, typically, for a large problem, 250 kHz or 5 channels. If a channel is to be used by a high power transmitter then its frequency separation should be larger, say 500 kHz or 10 channels. The constraint can therefore be of the form:

$$|f_i - f_j| \geq m$$

where m refers to the number of channels separation required between transmitters i and j.

Other types of constraints that can arise include *intermodulation products* and *spurious emissions and responses.* However, in this paper we only consider co-site frequency separation constraints, and co-channel and adjacent channel constraints, as these constraints represent the most important interference problems to avoid. Also, we have assumed that interference can be avoided if there is sufficient channel separation between the frequencies assigned to pairs of transmitters.

1.2 Sequential Assignment Algorithms

Sequential assignment methods mimic the way the problem might be solved manually. They are fast enough for large problems but tend to give results which are well short of the best possible. The transmitters are simply considered one at a time, successively assigning allowable frequencies as we proceed, until either we have assigned all transmitters or run out of frequencies. An important factor affecting the quality of solutions generated by this method is how the next transmitter is chosen. In addition, the initial ordering of the transmitters is important, as is the method by which the next frequency is chosen. We may therefore generate a series of assignment methods based on three components:

- initial ordering,
- choice of next transmitter,
- assignment of frequency.

The simplest way to choose the next transmitter is sequentially, simply picking the next one on the list produced by the initial ordering. A more complicated method, which has proved more effective than sequential selection with the various initial ordering methods, is called ***generalised saturation degree.*** In this method the choice of the next transmitter is influenced by the constraints imposed by all those transmitters that have already been chosen. One could view the more complicated process as a mechanism for correcting those mistakes that have been made by the initial ordering technique.

The simplest assignment technique is to assign the selected transmitter to the smallest acceptable channel i.e. the lowest numbered channel to which it can be assigned without violating any constraints. Variations upon this technique attempt to assign transmitters to channels that are already used in favour of those that are not. A detailed description of sequential assignment methods can be found in [10].

In this paper a genetic algorithm is used to search the state-space of initial orderings. The choice of the next transmitter is made sequentially, and the smallest acceptable channel is assigned to each chosen transmitter.

2 The Genetic Algorithm

A simple genetic algorithm (GA) which appeared in [12] is used for this work. It is derived from the model of [7] and is an example of a 'steady state' GA (based on the classification of [11]). It uses the 'weaker parent replacement strategy' first described by [3]. The GA applies the genetic operators to permutations of transmitters. The fitness values are based on the spans produced when the simple sequential assignment algorithm is applied to each permutation list produced by the GA. The first parent was selected deterministically in sequence, and the second parent was selected in a roulette wheel fashion, the selection probabilities for each genotype being calculated using the following formula:

$$\begin{array}{c}\text{selection}\\ \text{probability}\end{array} = \frac{(\text{population size} + 1 - \text{Rank})}{\sum \text{Ranks}}$$

where the genotypes are ranked according to the values of the spans that they have produced, with the best ranked 1, the second best 2 etc.

Mutation: The mutation chosen was to select two transmitters at random from a permutation list, and swap them.

Table 1: Test data characteristics.

	No. transmitters	No. co-site constraints	No. far-site constraints	edge density
test12	12	16	21	0.56
test95	95	90	1124	0.27
test190	190	160	4882	0.28
test410	410	411	22346	0.27
hex481	481	0	97835	0.85

Permutation Crossovers: Permutation crossovers were originally developed primarily for the travelling salesman problem (TSP), where the genotypes consist of lists of cities which are converted to TSP tours. Because TSP tours are circuits, it is irrelevant which city is represented first on the list. The permutation lists represent cycles and an edge in a TSP tour always joins the last city on the list to the first. Thus, for the TSP it is the relative sequence of cities that is important, rather than the absolute sequence. In the frequency assignment problem (FAP), however, the permutation lists making up the genotypes represent lists of transmitters, and intuitively it would seem likely that absolute sequences are important in this case.

The best known permutation operators from an historical standpoint (which are also amongst the simplest to implement) are partially matched crossover (PMX), order crossover (OX) and cycle crossover (CX) [5]. PMX, OX and CX are the chosen crossovers for this study on the FAP. Although many more sophisticated variations of permutation crossover have been developed over the years and these have proven far more successful on the TSP (for example, genetic edge recombination [13] and maximum preservative crossover [6]), unfortunately these variations rely almost entirely on problem-specific heuristics for their improved performance.

Pearson's correlation coefficient was used to assess how effective the various permutation crossovers were at propagating parental qualities to the offspring. Mid-parental values of "span" were correlated with the values for their respective offspring for a 95 transmitter problem. From an initial population of 1000 individuals, 1000 pairs of parents were selected, and from them 1000 offspring were generated using one of PMX, OX or CX and no mutation. The values of the correlation coefficients for all three permutation crossovers proved to be highly significant at the 0.0001% level, with the value for CX the best (0.3937).

3 Results

The test problems used are given in Table 1. The files test*xxx* refer to military frequency assignment problems that arise in irregular networks. The file hex481 refers to a cellular assignment problem which is loosely based on a regular network arising around the Philadelphia area in the USA [1].

Results based on single runs of the GA are presented in Table 3. The GA is compared with the best result obtained from using several state-of-the-art sequential assignment algorithms [10]. The cosite value refers to the value of m used for the cosite frequency separation constraint (see section 1.1).

The 'optimum' column refers to the known optimum span (where available). (The optimum values are obtained either from using exhaustive search techniques where the problem is small enough, or where computed results match theoretical lower bounds [9]). The 'random search' column corresponds to the best span produced by random search through permutation space for each of the problems, where each random search processed exactly the same number of individuals as the corresponding GA.

We can see that the GA, which starts with a population of random permutations of transmitters, produces an improvement over the results from the sequential assignment algorithms, and reaches the

Table 2: Timing results.

Problem	Number of constraints	Time (secs)	Time / # constraints
test12	37	30	0.81
test95	1214	518	0.43
test190	5042	2765	0.55
test410	22757	20019	0.88
hex481	97835	229916	2.35

Table 3: Comparison of results.

Problem	GA			Best Sequential	Random Search	Optimum
	PMX	OX	CX			
test12	22	22	22	24	22	22
test95 (cosite 4)	48	48	48	51	50	48
test95 (cosite 5)	48	49	48	54	52	48
test190	82	84	76	87	84	–
test410	165	165	154	158	170	–
hex481	443	442	426	449	475	426

optimum value in several cases. The run times for the test examples are given in Table 2. In each case the GA was run using a population of 200 for 200 generations.

4 Concluding Remarks

A genetic algorithm has been presented which finds an initial ordering of the transmitters. Each transmitter in this ordered set is then assigned the smallest frequency possible while still satisfying any associated constraints. Of the permutation crossover operations used, cycle crossover gives the best results in all the test cases and gives the optimum solution in four out of six cases. Further work is necessary to test the applicability of the technique to larger problems, and to develop efficient problem specific crossover operators.

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