A Hierarchical Solve-and-Merge Framework for Multi-Objective Optimization

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Introduction

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- Stage 1 consists of subpopulations running single objective GAs
- Each with its own uniquely weighted linear scalarizing function to focus on a different region of Pareto space
- Stage 2 collects the best solutions from stage 1 and runs a hierarchical Pareto-based EMO

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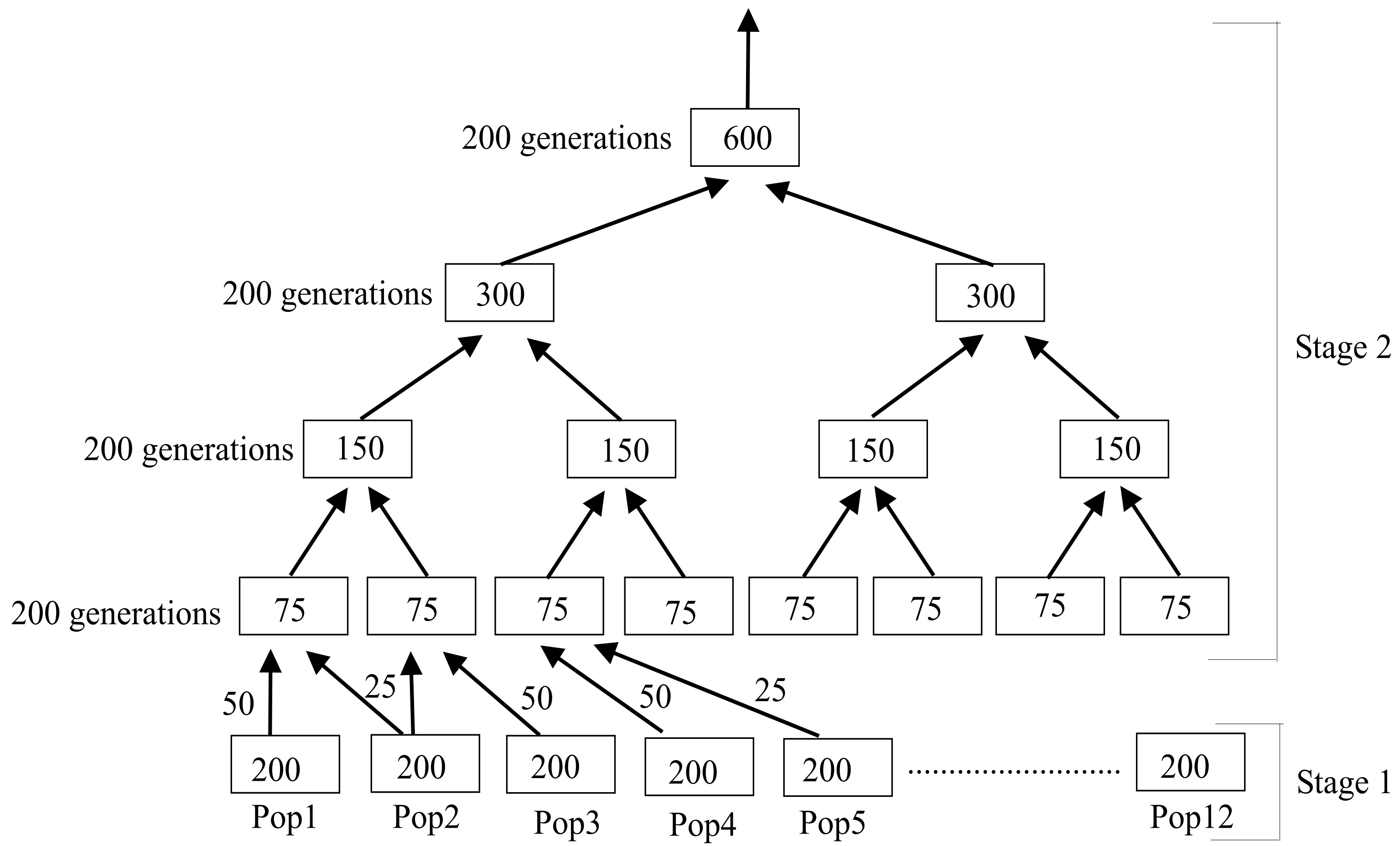
The two-stage framework of HISAM



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Stage 2

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- Using subpopulations and a hierarchy produced a much better spread of solutions than SEAMO2 on a single large population
- The addition of the single objective stage 1 forces each subpopulation to focus on a different region of Pareto Space

Stage 1

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- Each copy runs on its own independent subpopulation
- Each having a differently weighted linear scalarizing function
- E.g. Objective function = weight1 X objective1 + weight2 X objective2 etc.
- The best individuals from each subpopulation are passed to stage 2

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Solution sets should be

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SEAMO algorithms good at (1) and (2), but not always at (3)

Why do we need stage 1?

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- Setting up multiple subpopulations with each covering the whole Pareto space would seem wasteful.
- Question: Can we improve results further by concentrating the efforts of the subpopulations more locally?

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Why do we need stage 2?

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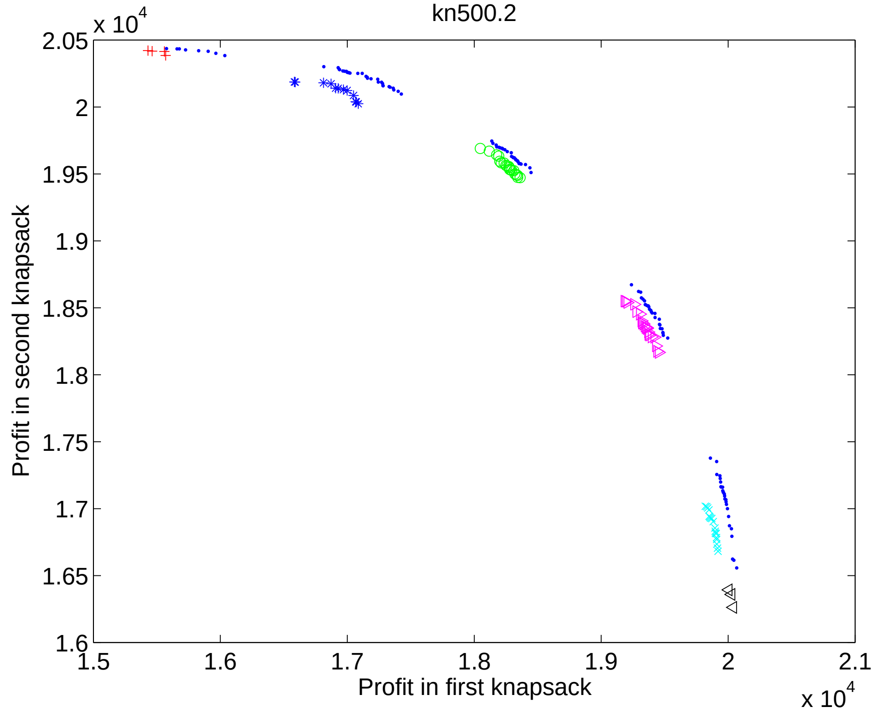

Single run of stage 1 with 6 subpopulation (lower), followed by a simplified stage 2. (upper). Without a hierarchical stage 2 we get discontinuities.

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The Simple GA (stage 1)

Procedure *SIMPLE GA*

Begin

Generate N random individuals {N is the population size}

Evaluate the fitness function for each population member and store it

Repeat

For each member of the population

This individual becomes the first parent

Select a second parent at random

Apply crossover to produce single offspring

Apply a single mutation to the offspring

Evaluate the objective function produced by the offspring

if the offspring is better than weaker parent (and not a duplicate)

Then the offspring replaces it in the population

else it dies

Endfor

Until stopping condition satisfied

End

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The SEAMO Framework (stage 2)

Procedure *SEAMO*
Begin
  Generate N random individuals {N is the population size}
  Evaluate the objective vector for each population member and store it
  Repeat
    For each member of the population
      This individual becomes the first parent
      Select a second parent at random
      Apply crossover to produce single offspring
      Apply a single mutation to the offspring
      Evaluate the objective vector produced by the offspring
      if offspring qualifies
        Then the offspring replaces a member of the population
      else it dies
    Endfor
  Until stopping condition satisfied
  Print all non-dominated solutions in the final population
End

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Replacement Strategy for SEAMO2

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- (2) Does offspring produce a global improvement on any Pareto components?
- (3) Does offspring have a mutually non-dominating relationship with both its parents?
- (4) Is offspring a duplicate?

Parameters for experimental runs

Table 1: Population parameters for experimental runs

Problem	Algorithm						
	HISAM					SEAMO2	
	stage 1			stage 2			
	# subpops	subpopsize	generations	popsize	generations	popsize	generations
kn500.2	12	200	200	600	800	960	1,000
kn750.2	12	250	250	600	1000	1080	1,250
kn750.3	15	250	300	750	1200	1350	1,500
kn750.4	35	250	400	1,750	1600	3150	2,000

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Representation for the MKP

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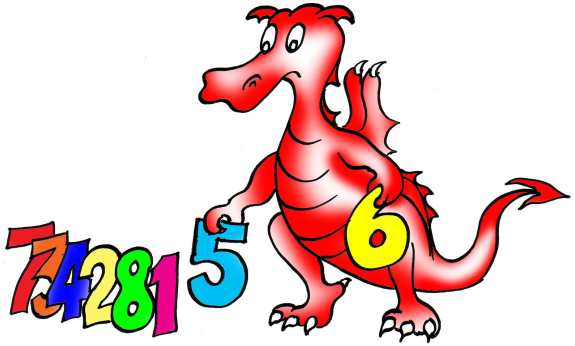

- Order-based representation with a first fit decoder

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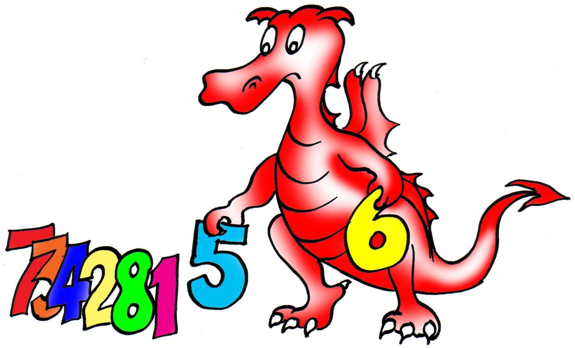

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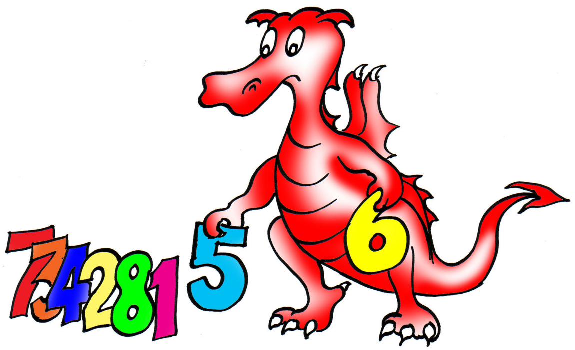

- Order-based representation with a first fit decoder
- Cycle Crossover (CX)
- A simple mutation operator swaps two arbitrarily selected objects within a single permutation list

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Experimental Method

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Experimental Method

- HISAM is evaluated against 2 benchmarks:
  1. SEAMO2
  2. MOGLS
- 30 replicate runs carried out for each set of experiments, and all algorithms use the same number of objective function evaluations

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Why SEAMO2 and MOGLS?

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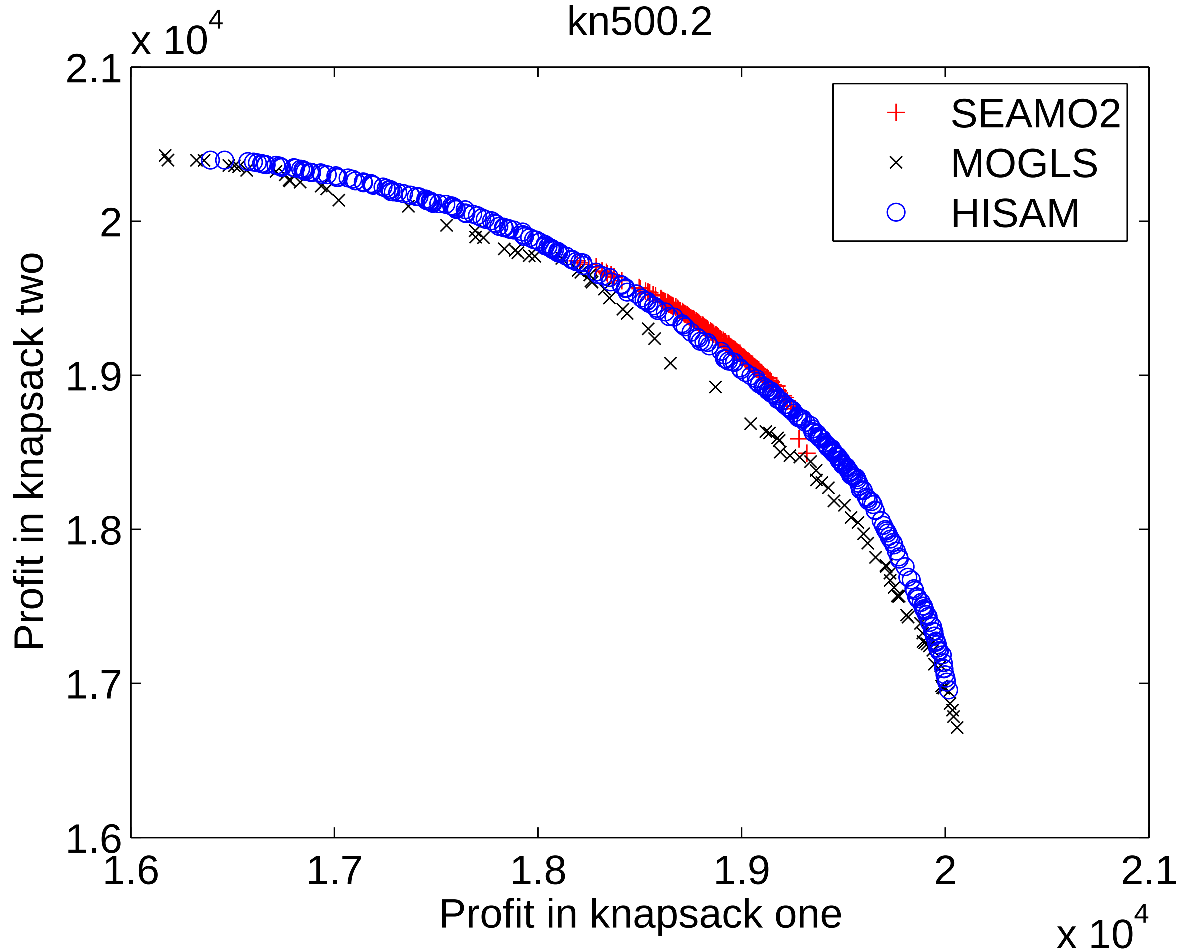
- HISAM is derived from SEAMO2
- MOGLS results are very hard to beat on the MKP

Results for Multiple Knapsack Problems



Example runs, comparing HISAM with MOGLS and SEAMO2 on kn500.2

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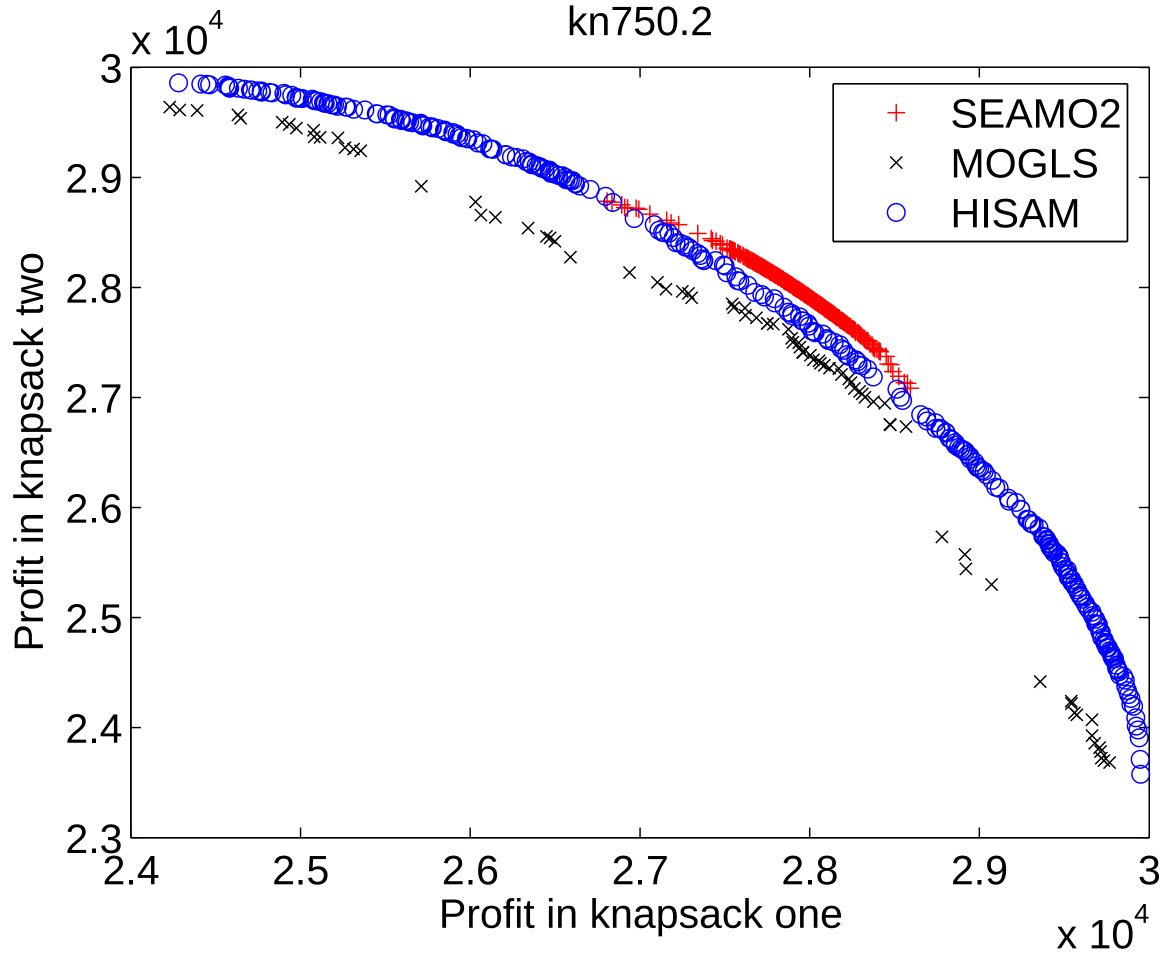
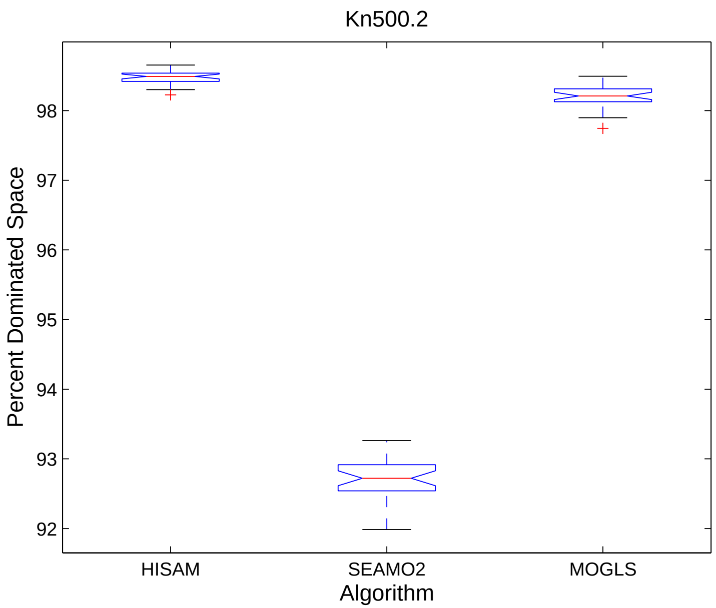
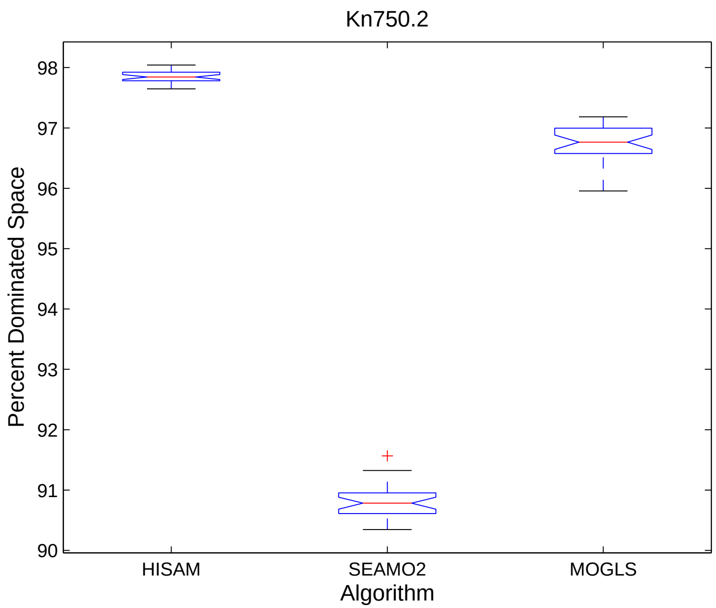


Example runs, comparing HISAM with MOGLS and SEAMO2 on kn750.2

Comparing Dominated Space





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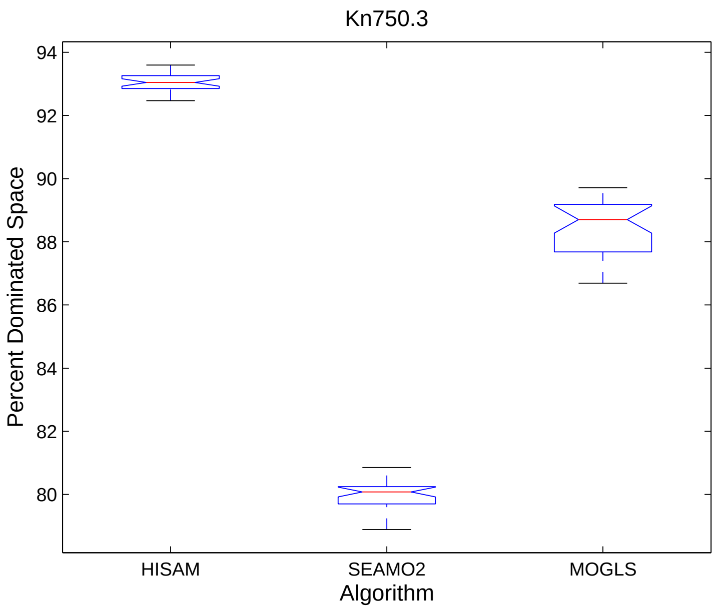

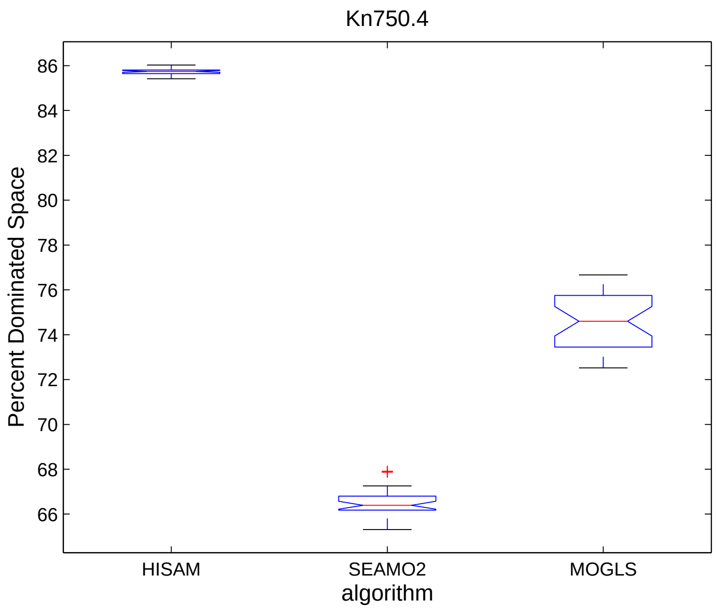

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Comparing run times of the multi-objective algorithms (in seconds)

Algorithm	kn500.2	kn750.2	kn750.3	kn750.4
HISAM	59	105	207	1,097
SEAMO2	139	254	572	4,208
MOGLS	2,918	8,955	14,085	45,099

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- It seems to be a great improvement on previous versions of SEAMO
- It outperforms MOGLS in terms of solution quality
- It runs very fast

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- Implement on parallel hardware

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