Qualitative Representation and Reasoning with Uncertainty in Space and Time

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Abstract

Imprecision, indeterminacy and vagueness are all terms which have been studied recently in studies of representations of entities in space and time. The interest has arisen from the fact that in many cases, precise information about objects in space are not available. In this paper a study of spatial uncertainty is presented and extended to temporal uncertainty. Different types and modes of uncertainty are identified. A unified framework is presented for the representation and reasoning over uncertain qualitative domains. The method addresses some of the main limitations of the current approaches. It is shown to apply to different types of entities with arbitrary complexity with total or partial uncertainty. The approach is part of a comprehensive research program aimed at developing a unified complete theory for qualitative spatial and temporal domains.

1 Introduction

The ability to handle a certain level of indeterminacy makes techniques of qualitative spatial reasoning (QSR) attractive to many application domains [Haa95, FM91, Liu69]. Precise information required in quantitative methods are sometimes neither available nor needed. For example, "vague" expressions of place names, locations and spatial relationships may be used in searches over large geographic databases, where in many cases exact positional information is not available or can't be expressed. Recently, the rapid development of wireless communication devices and sensor networks enabled the emergence of a wide variety of applications that require efficient access to and management of dynamic spatio-temporal data. In many such applications, data are generated at rapid rates and accepting " approximate" data is a strategy for reducing both data size and associated costs. Recently, there has been an upsurge on explicit representation of imprecise and indeterminate regions [CDF97]. Current approaches to representation and reasoning are mostly limited to handling simple entities in both the space and time dimensions. Proposals are generally extensions of existing approaches for representation in spatial and teporal domains and therefore tend to carry their limitations.

This paper starts with a study of the notion of qualitative uncertainty. Possible types and modes of uncertainty are identified. A uniform model of representation in uncertain spaces and time is then presented and examples are used to demonstrate its validity for random object types. A reasoning mechanism is proposed and applied for the composition of relations.

Types of Uncertainty Representation and reasoning formalisms for handling "crisp" objects have been proposed previously. In this work we introduce the different types of uncertainty that may exist in space and time. The 1d temporal domain is considered as a special case of the much richer 3D space. Hence, in what follows we provide a general view of uncertainty in space. Different spatial attributes can be associated with an object in 3D space to define, for example, its position, shape, configuration, orientation, etc. The accuracy of the representation of the object is directly dependent on the values of those attributes. To precisely define a spatial object, each of its associated properties must hold a unique value. However, this value may be one of a number of possible and correct values that can be associated with a spatial property. For example, Eiffel Tower as a place could be defined to be in Europe, in France, in Paris, or can be described exactly by its (x,y) map grid reference. Hence, spatial uncertainty of objects

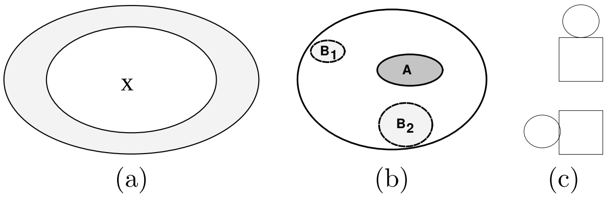


Figure 1: (a) Object extension uncertainty. (b) Configuration uncertainty. (c) Object orientation uncertainty

in space occurs when one or more spatial attribute associated with an object holds more than one of a set of possible values.

Different types of uncertainty can be defined as follows.

Positional uncertainty: where the precise location of an object or one of its constituting components is not certain. An example from the temporal domain is: "John came back from holiday last month".

Extension uncertainty: where the extent of the object's boundary or the boundary of one of its components is not certain, as shown in figure 1(a). The shaded ring in the figure represents the area within which the boundary may be found. This type of uncertainty is also applicable in the temporal domain.

Configuration uncertainty: where the specific components making up a composite spatial object and their number are not certain. Figure 1(b) shows a region with holes where it is not known whether the component holes are A and B_1 or A and B_2. An example of this type of uncertainty in the temporal domain is: Activity A should overlap or occur during activity B.

Orientation uncertainty: where the orientation of the object, or the orientation of one of its components is not certain as shown in figure 1(c). This type of uncertainty is not applicable in the 1D directed temporal domain.

Modes of Uncertainty

To illustrate the different modes of uncertainty, examples from the temporal domain are used, for the sake of simplicity of its one dimensional nature. Two modes of uncertainty can be distinguished, namely, discrete and range. An example of discrete uncertainty is stating that, "I will arrive at either 10 am or 11 am". An example of range uncertainty, is when the arrival time is defined by a range of ordered values, for example, "I will arrive between 10:00 am and 11:00 am". Examples of those modes in space are used later on in the paper. Note that all types of uncertainty listed above can exist in both the discrete and range modes.

2 Related work

Three possible models for representing uncertainty in space and time are fuzzy models, probabilistic models and exact models [ES97, BG00, BTM02, CM02, DFP96]. With exact models, "existing definitions, techniques, data stuctures, algorithms need not be redeveloped but modified or simply used .. " [ES97]. "Exact" approaches are generally based on one of two models of representation in space, namely, the Region-Connection-Calculus (RCC) [CG94, HC01, RS01, CDF97, CDF96b, CG94] and the Intersection-based approach [TN, Zha98, TJ, Sch01]. They deal mainly with simple convex regions and with range uncertainty over the boundary of those regions. In the temporal domain, most approaches are based on the work of Allen **??**.

In the spatial domain the "egg and yolk" approach [CG96] uses an analogy for defining objects, where the difference between the egg and the yolk represents a range of uncertainty of the object's boundary. Different sets of relations have been identified for those objects; 46 relations in [CG96] using first order logic and 44 relations using the intersection-based approach [CDF96a]. Clementini et al [CDF97] added a further set of 12 relations for representing composite objects with indeterminate boundaries.

Other variations of the above methods have also been used to define the set of relations by considering the egg and yolk as crisp objects and then considering the combinatoric set of relations between them. In [HC01], the changes in the egg and yolk were considered for the purpose of defining a spatio-temporal interpretation of the method, for example, by noting the increase, decrease or the stability of the egg and yolk respectively.

In the fuzzy approaches, [TN] used an aggregate uncertainty with values of 1.0 and 0.5 concentric regions of core and support to define the set relations as above. In [Zha98], the fuzzy set was divided up into more than 2 concentric regions with values between 1 and 0. Some works have also addressed the definition of fuzzy complex regions [Sch01] and a degree of belief is assigned based on a ratio of representation of the characteristic feature, e.g. the area of overlap to the area of one of the objects. Applications of fuzzy relations has been demonstrated in [TJ] in the domain of guiding autonomous vehicle motion.

It is worth noting here that as far as known, no work in space has been reported on the other types of uncertainty, namely, orientation and spatial arrangements. Modes of uncertainty were mainly confined to range uncertainty and no work has addressed the discrete mode of uncertainty. Also, no methods have yet been proposed to the composition of relations in uncertain space.

3 Representation of Uncertainty of object Properties

An exact modelling approach is adopted here. In this section a representation scheme for the different types and modes of spatial and temporal uncertainty is presented. The method is based on and extends the approach proposed in [BA97] for representing crisp spatial and temporal objects. Figure 2(a) is an example of the representation of a simple object using this method. Objects of interest and their embedding space are divided into components according to a required resolution. The connectivity of those components is explicitly represented. In the decomposition strategy, the complement of the object in question is considered to be infinite, and the suffix 0, e.g. (x_0) is used to represent this component. Object x in figure 2(a) is composed of three components, namely, x_1; an areal component representing the object's interior, x_2; a linear component representing the object's boundary, and x_0, the rest of the embedding space. In what follows, examples of representation of objects with spatial uncertainty are given.

3.1 Representation of Object Location uncertainty

Discrete location uncertainty can be represented by placing a copy of the object in each of the possible values of the uncertain location, as shown in figure 2(b) for spatial and temporal objects respectively. In the figure, object x is represented by both copies $x^{'}$ and $x^{"}$. Using the representation scheme as above, the space containing both copies is represented by the intersection of their respective components. The intersection of $x^{'}$ and $x^{"}$ is a definite part of the component x_1. The rest of both $x^{'}$ and $x^{"}$ can be either x_1 or x_0, and hence is labelled $(x_1 \vee x_0)$. Similarly, different parts of objects' boundaries can be labelled according to whether their comprising points are possibly part of either x_1 or x_2 or x_0 as shown in figure 2(c).

If the locations of $x^{'}$ and $x^{"}$ represents the bounds of a range uncertainty as shown in figure 2(d), the representation changes to include the boundary as one of the possible components inside the two circles and between the circles and their tangents as shown. Also, the two points representing certain points on the boundary x_2 do not exist any longer.

3.2 Representation of object extension uncertainty

A decomposition scheme in the case of range uncertainty is shown in figure 3(a) where the boundary x_2 of x can exist anywhere between $(x_1 \vee x_2)$ and $(x_2 \vee x_0)$. It is interesting to note that all related work on uncertainty of space [] was concerned mainly with range uncertainty over simple convex regions. Note that if the boundaries of vagueness, i.e. $(x_1 \vee x_2)$ and $(x_0 \vee x_2)$ are not known (opensets) then uncertainty

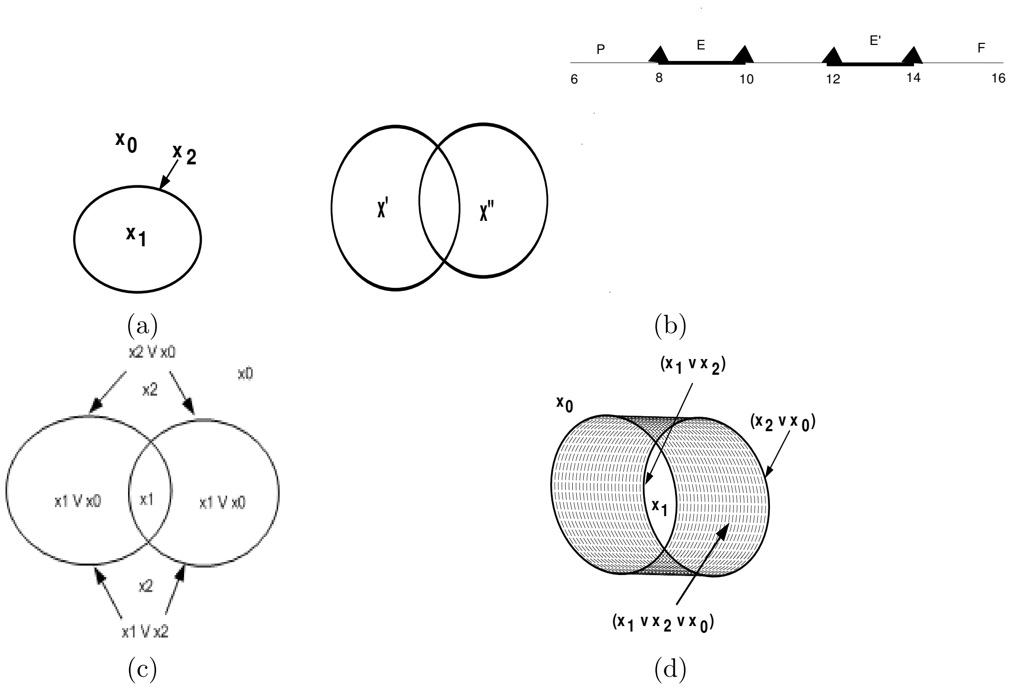


Figure 2: (a) Topology of objects represented by components. (b) and (c) Discrete location uncertainty. (d) Range uncertainty.

in this case is represented as in figure 3(b) where the limiting boundaries are omitted and the object is represented by two components, x_1 and x_0. This representation is used in [CG96, CDF96a].

An example of discrete uncertainty of object's extension is shown in figure 3(c) where the boundary can exist either at $(x_1 \vee x_2)$ or $(x_2 \vee x_0)$. The area in-between must be either x_1 or x_2.

In the case of a combined location and extension uncertainty the representation in figure 3(d) is used where there are no assigned regions for x_1 only.

Partial uncertainty in the case of convex region is represented as in figure 4(a) where part of the boundary is definite (x_2) and the rest is bounded between $(x_1 \vee x_2)$ and $(x_2 \vee x_0)$. A similar partial range uncertainty in case of concave object is shown in figure 4(b) where the mode is discrete partial uncertainty since it's only the boundary of concavity which is under uncertainty.

4 Temporal Uncertainty

The approach presented in this paper can be used to represent uncertainty in the temporal domain. In figure 8(a), a temporal interval is shown embedded in a directed on-dimensional space, with two semi-infinite lines representing the past P and future F of the interval. The interval itself is decomposed into three components, two points s and e, representing its start and end and an open line d representing its duration.

Different types of uncertainty can be represented in a similar fashion as before. For example, a temporal interval with uncertainty over its start and end states is shown in figure 8(b). An example of this case is: "the event will start between 9 and 10 and ends between 1 and 2". Similarly , figure 8(c) is an example of partial discrete uncertainty. Here, the case represented is: "the event will start between 9 and 10 and ends at 1".

The reasoning method proposed can therefore be extended homogeneously to the temporal dimension. Finding a common approach for the representation and reasoning over space and time provides an opportunity for integrating of the temporal dimension in the management of spatial data.

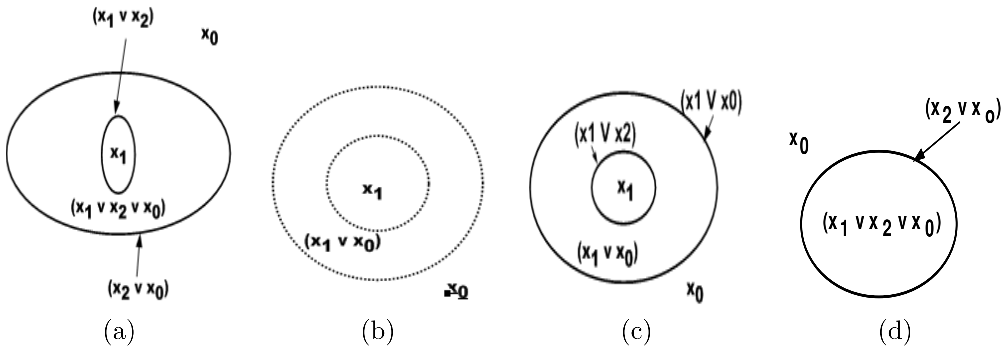


Figure 3: (a), (b), (c) Extent-Discrete uncertainty. (d) Combined location and extension uncertainty

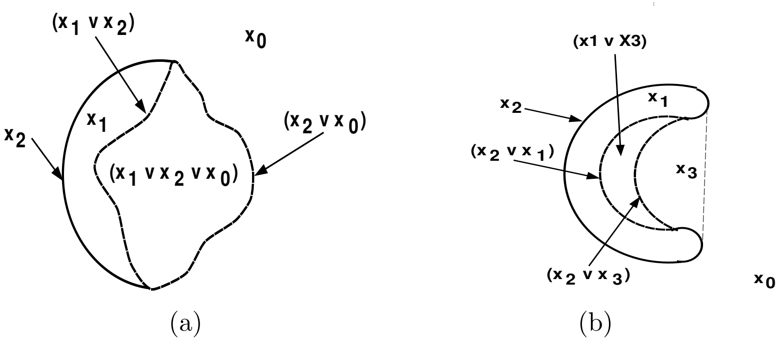


Figure 4: (a) Partial Extent uncertainty. (b) Partial Extent-Range uncertainty.

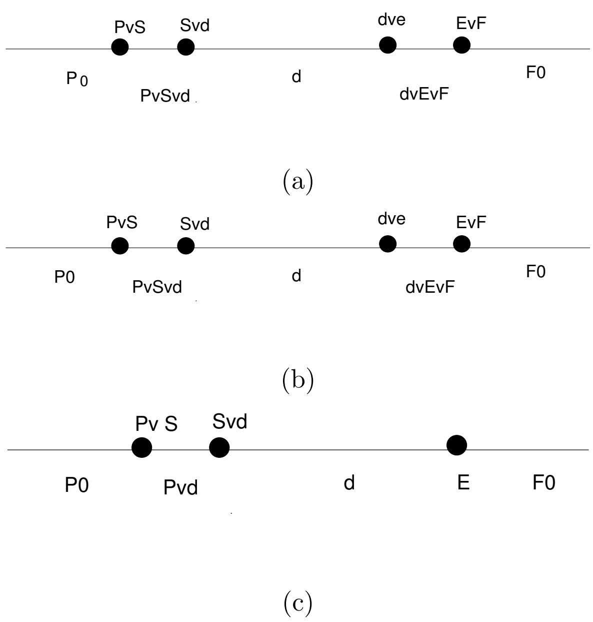
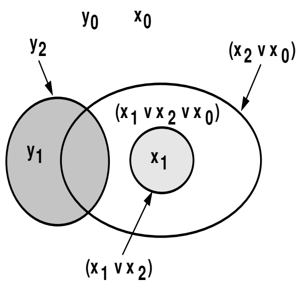


Figure 5: (a) A temporal interval. (b) and (c) Example of temporal intervals with uncertainty.



(a)

	y_1	y_2	y_0
x_1	0	0	1
$x_1 \vee x_2$	0	0	1
$x_1 \vee x_2 \vee x_0$	1	1	1
$x_2 \vee x_0$	1	1	1
x_0	1	1	1

(b)

Figure 6: (a) Relationship between a range-uncertain object x and a crisp object y (b) Corresponding intersection matrices.

5 Representation of Spatial Relations in Uncertain Spaces

In this section, the representation of the topological relations through the intersection of their components is adopted and generalized for objects with spatial uncertainty. The complete set of spatial relationships are identified by combinatorial intersection of the components of one space with those of the other space.

If $R(x, y)$ is a relation of interest between objects x and y, and X and Y are the spaces associated with the objects respectively such that m is the number of components in X and l is the number of components in Y, then a spatial relation $R(x, y)$ can be represented by one instance of the following equation:

$$\begin{aligned} R(x,y) &= X \cap Y \\ &= \left(\bigcup_{i=1}^{m} x_i \right) \cap \left(\bigcup_{j=1}^{l} y_j \right) \\ &= (x_1 \cap y_1, \cdots, x_1 \cap y_l, x_2 \cap y_1, \cdots, x_m \cap y_l) \end{aligned}$$

The intersection $x_i \cap y_j$ can be an empty or a non-empty intersection.The above set of intersections shall be represented by an intersection matrix, as follows,

$R(x, y) =$

	y_0	y_1	y_2	$\cdots$
x_0				
x_1				
x_2				
$\vdots$				

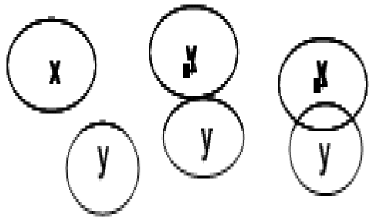
Different combinations in the intersection matrix can represent different qualitative relations. The set of valid or sound spatial relationships between objects is dependent on the particular domain studied.

Example: Range Extent Uncertainty Relations

Consider the relationship between objects x and y in figure 6(a). Object x is spatially vague, with range uncertainty mode. Object y is crisp. The intersection matrix representing the relationship is shown in 6(b). The intersection matrix can be rewritten by mapping the components uncertainty into intersections relation uncertainty between crisp objects in figure 7(a). On comparing the matrix with the set of of 8 relations between two simple crisp regions, a different representation of the relation in 6(a) can be described as a disjunctive set of relations $\{disjoint \vee touch \vee overlap\}$ as shown in figure 7(b).

	y_1	y_2	y_0
x_1	$0 \vee 1$	$0 \vee 1$	1
x_2	$0 \vee 1$	$0 \vee 1$	1
x_0	1	1	1

(a)



(b)

Figure 7: (a) Mapped intersection relation. (b) Possible relations.

5.1 Representation of Temporal Relations in Uncertain Space

In the above section it was shown how range and discrete uncertainty can be represented using disjunctive sets of components. In this section, we consider the representation of relations between two uncertain events where the uncertainty is discrete and partial.

Consider the following scenario: "John arrived at the party between 7pm and 8pm and left at 11:00 pm. Alice arrived either before or after John and left at 10:30 pm.". Figure 8(a) depicts this scenario in graphical form and the intersection matrix of the temporal relations is shown in figure 8(b).

6 Reasoning with Uncertainty

The reasoning approach consists of: a) general constraints to govern the spatial relationships between objects in space, and b) general rules to propagate relationships between the objects.

6.0.1 General Reasoning Rules

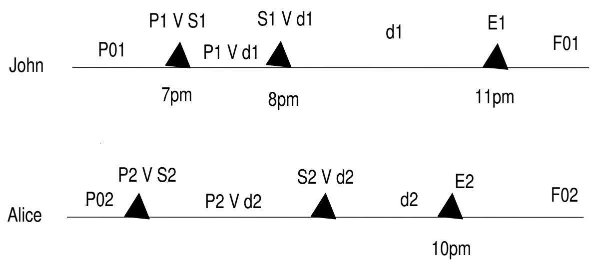
Composition of spatial relations is the process through which the possible relationship(s) between two object x and z is derived given two relationships: R_1 between x and y and R_2 between y and z. Two general reasoning rules for the propagation of intersection constraints are presented. The rules are characterized by the ability to reason over spatial relationships between objects of arbitrary complexity in any space dimension. These rules allow for the automatic derivation of the composition (transitivity)tables between any spatial shapes.

Rule 1: Propagation of Non-Empty Intersections

Let $x' = \{x_1, x_2, \cdots, x_{m'}\}$ *be a subset of the set of components of space* X *whose total number of components is* m *and* $m' \leq m$*;* $x' \subseteq X$*. Let* $z' = \{z_1, z_2, \cdots, z_{n'}\}$ *be a subset of the set of components of space* Z *whose total number of components is* n *and* $n' \leq n$*;* $z' \subseteq Z$*. If* y_j *is a component of space* Y*, the following is a governing rule of interaction for the three spaces* X*,* Y *and* Z*.*

$$
\begin{aligned}
(x' \sqsupseteq y_j) \quad \wedge \quad & (y_j \sqsubseteq z') \\
\rightarrow \quad & (x' \cap z' \neq \phi) \\
\equiv \quad & (x_1 \cap z_1 \neq \phi \vee \cdots \vee x_1 \cap z_{n'} \neq \phi) \\
& \wedge (x_2 \cap z_1 \neq \phi \vee \cdots \vee x_2 \cap z_{n'} \neq \phi) \\
& \wedge \cdots \\
& \wedge (x_{m'} \cap z_1 \neq \phi \vee \cdots \vee x_{m'} \cap z_{n'} \neq \phi)
\end{aligned}
$$

The above rule states that if the component y_j in space Y has a positive intersection with every component from the sets x' and z', then each component of the set x' must intersect with at least one component of the set z' and vice versa.

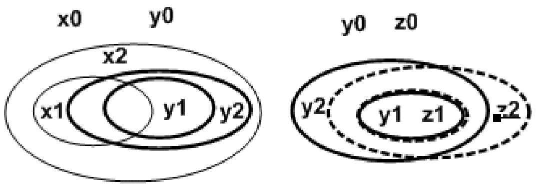


(a)

	P_{01}	$P_1 \vee S_1$	$P_1 \vee d_1$	$S_1 \vee d_1$	d_1	E_1	f_{01}
P_{01}	1	0	0	0	0	0	0
$P_2 \vee S_2$	1	0	0	0	0	0	0
$P_2 \vee d_2$	1	1	1	1	1	0	0
$s_2 \vee d_2$	0	0	0	0	1	0	0
d_2	0	0	0	0	1	0	0
E_2	0	0	0	0	1	0	0
f_{02}	0	0	0	0	1	1	1

(b)

Figure 8: (a) Graphical representation of sample uncertain temporal events. (b) Corresponding intersection matrix.



	y_1	y_2	y_0
x_1	1	1	1
x_2	1	1	1
x_0	0	0	1

(c)

	z_1	z_2	z_0
y_1	1	0	0
y_2	0	1	1
y_0	0	1	1

(d)

Figure 9: (a) and (b) Spatial relationships between vague regions x, y and z. (c) and (d) Corresponding intersection matrices.

The constraint $x_i \cap z_1 \neq \phi \ \vee \ x_i \cap z_2 \neq \phi \cdots \ \vee \ x_i \cap z_{n'} \neq \phi$ can be expressed in the intersection matrix by a label, for example the label a_r ($r = 1$ or 2) in the following matrix indicates $x_1 \cap (z_2 \cup z_4) \neq \phi$ (x_1 has a positive intersection with z_2, or with z_4 or with both). A $-$ in the matrix indicates that the intersection is either positive or negative.

	z_1	z_2	z_3	z_4	$\cdots$	z_n
x_1	$-$	a_1	$-$	a_2	$-$	$-$

Rule 1 represents the propagation of non-empty intersections of components in space. A different version of the rule for the propagation of empty intersections can be stated as follows.

Rule 2: Propagation of Empty Intersections

Let $z' = \{z_1, z_2, \cdots, z_{n'}\}$ be a subset of the set of components of space Z whose total number of components is n and $n' < n$; $z' \subset Z$. Let $y' = \{y_1, y_2, \cdots, y_{l'}\}$ be a subset of the set of components of space Y whose total number of components is l and $l' < l$; $y' \subset Y$. Let x_i be a component of the space X. Then the following is a governing rule for the spaces X, Y and Z.

$$\begin{aligned} (x_i \sqsubseteq y') \quad & \wedge \quad (y' \sqsubseteq z') \\ & \rightarrow \quad (x_i \cap (Z - z_1 - z_2 \cdots - z_{n'}) = \phi) \end{aligned}$$

6.1 Example: Reasoning with Range Extent Uncertainty

In [CDF01], Clementini et al defined a set of 44 possible relations between objects with undetermined boundaries (range extension uncertainty). In this example, we use two of those relations, shown in figure 9, to demonstrate the composition of spatial relationships in uncertain spaces. In [CDF01] objects were represented using three components, of boundary, interior and exterior. Using the proposed representation methodology above, objects are represented by the three components: $\{x_1, (x_1 \vee x_0), x_0\}$ as shown in figure 3(b), where the broad boundary is represented by the disjunctive set of possible components. In this example, x_2 is used to represent $(x_1 \vee x_0)$.

The reasoning rules are used to propagate the intersections between the components of objects x and z as follows. From rule 1 we have,

- y_1 intersections:

$$\begin{aligned} \{x_1, x_2\} \sqsupseteq y_1 \quad & \wedge \quad y_1 \sqsubseteq \{z_1\} \\ & \rightarrow \quad x_1 \cap z_1 \neq \phi \wedge x_2 \cap z_1 \neq \phi \end{aligned}$$

- y_2 intersections:

$$\begin{aligned} \{x_1, x_2\} \sqsupseteq y_2 \quad & \wedge \quad y_2 \sqsubseteq \{z_2, z_0\} \\ & \rightarrow \quad x_1 \cap (z_2 \cup z_0) \neq \phi \wedge x_2 \cap (z_2 \cup z_0) \neq \phi \end{aligned}$$

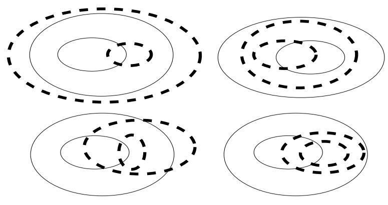


Figure 10: Possible configurations for the composition in figure 8.

- y_0 intersections:

$$\begin{aligned} \{x_1, x_2, x_0\} \sqsupseteq y_0 \quad & \wedge \quad y_0 \sqsubseteq \{z_2, z_0\} \\ & \rightarrow \quad x_1 \cap (z_2 \cup z_0) \neq \phi \wedge x_2 \cap (z_2 \cup z_0) \neq \phi \\ & \wedge \quad x_0 \cap (z_2 \cup z_0) \neq \phi \end{aligned}$$

Refining the above constraints, we get the following intersection matrix.

	z_1	z_2	z_0
x_1	1	a_1,c_1	a_2,d_1
x_2	1	b_1,c_2	b_2,d_2
x_0	0	?	1

Where a_1 and a_2 represent the constraint $x_1 \bigcap (z_2 \vee z_0) = 1$ and b_1 and b_2 represent the constraint $x_2 \bigcap (z_2 \vee z_0) = 1$, c_1 and c_2 represent the constraint $z_2 \bigcap (x_1 \vee x_2) = 1$ and d_1 and d_2 represent the constraint $z_0 \bigcap (x_1 \vee x_2) = 1$ and the ? represents $(1 \vee 0)$. The result matrix corresponds to one of four possible relationships between x and z, namely numbers 21, 22, 23 and 25, as shown in 10.

Reasoning with temporal uncertainty is handled in a similar way to reasoning in the spatial domain above, where spatial components are substituted with components in the temporal domain.

7 Conclusions

In this paper, uncertainty in space and time is studied. Four types of spatial uncertainty were identified, related to the different spatial properties of objects, namely, positional, extension, configuration and orientation. The first three types are applicable in the temporal domain. The concept of the "mode" of uncertainty was also introduced. Spatial and temporal uncertainty operates in two different modes, namely, discrete and range. Related approached have addressed the range uncertainty mode and were generally limited to handling simple object types, such as convex regions and intervals. No work has addressed the problem of reasoning with uncertainty. In this paper, an exact approach to the representation of uncertain space is proposed. The model is flexible and handles the different types and modes of uncertainty homogeneously. The approach can also be used is situations of partial uncertainty of objects and relations. The representation method is complemented with a general reasoning formalism to propagate different types of relations in uncertain spaces. Extending of the method to the temporal dimension has also been demonstrated. Work is in progress for further developing the methods and their realisation for spatio-temporal domains.

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