Deriving Extensional Spatial Composition Tables

Baher El-Geresy, Alia I. Abdelmoty and Andrew J. Ware

Abstract Spatial composition tables are fundamental tools for the realisation of qualitative spatial reasoning techniques. Studying the properties of these tables in relation to the spatial calculi they are based on is essential for understanding the applicability of these calculi and how they can be extended and generalised. An extensional interpretation of a spatial composition table is an important property that has been studied in the literature and is used to determine the validity of the table for the models it is proposed for. It provides means for consistency checking of ground sets of relations and for addressing spatial constraint satisfaction problems. Furthermore, two general conditions that can be used to test for extensionality of spatial composition tables are proposed and applied to the RCC8 composition table to verify the allowable models in this calculus.

1 Introduction

Qualitative Spatial Reasoning (QSR) is concerned with the qualitative aspects of representing and reasoning about spatial entities. The challenge of QSR is to "provide calculi which allow a machine to represent and reason with spatial entities of higher dimension, without resorting to the traditional quantitative techniques prevalent in, for example, the computer graphics or computer vision communities" [3]. There are many possible applications of QSR, for examples, in Geographical Information Systems (GIS), spatial query languages, natural languages and many other fields.

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One of the most widely referenced formalisms for QSR is the Region Connection Calculus (RCC), initially described in [11, 12] and intended to provide a logical framework for spatial reasoning.

Given a fixed vocabulary of relations, *Rels*, the composition table allows the answer of the following question by simple lookup: given two relational facts of the forms $R(a,b)$ and $S(b,c)$, what are the possible relations (from the set *Rels*) that can hold between a and c? Composition tables are essential tools for solving spatial constraint satisfaction problems, for example checking the integrity of a database of atomic assertions (involving relations in some set for which we have a composition table) by testing whether every three relations are consistent with the table [3].

An extensional interpretation of composition tables (CT) is a property that checks if the table is valid for the models it is associated with. The RCC theory allows regions in topological space as models with no restriction on their complexity. Previous research work have raised questions on the extensionality of the RCC CT and admitted that an extensional interpretation of the table is not compatible with the RCC theory [2, 1]. The question of when would a composition table have an extensional interpretation needs to be addressed. Answers to this question will allow for deeper understanding of current spatial calculi and their further development. The property of extensionality is a reverse interpretation of the property of composition. For example, one of the possible relations resulting from the composition $EC(touch)(a,b) \wedge EC(b,c)$ is the relation $EC(a,c)$. For extensionality to be achieved for any configuration of the relations $EC(a,c)$, there must exist a third object b such that both relations $EC(a,b)$ and $EC(b,c)$ exist.

In this paper, different type of regions that are possible models of the RCC theory are first identified and the extensionality problem is expressed in terms of the composition triads over the different combinations of those regions. Two general conditions are proposed that can be used to identify non-extensional cases in spatial composition. These are based on the property of connectivity of objects and space. The value of using these conditions are demonstrated through the application of a representation and reasoning approach that encapsulates the explicit representation of connection and hence always results in extensional compositions. Section 2 gives a brief survey of related work. In section 3, the extensionality problem is expressed. In section 4, the two general conditions for checking extensionality are proposed. Application of the proposed conditions is presented in section 6 on the defined extensionality problem. Summary and conclusions are given in section 6.

2 Related Work

RCC8 [3] is a topological constraint language based on eight atomic relations between extended regions of a topological space. Regions are regular subsets of a topological space, they can have holes and can consist of multiple disconnected pieces. The eight atomic relations DC (disconnected), EC (externally connected), PO (partial overlap), EQ (equal), TPP (tangential proper part),NTPP (non-tangential proper part) and their converse relations TPPi,NTPPi were originally defined in first-order logic. In this theory regular closed regions are considered, i.e., regions that are equivalent to the closure of their interior. The RCC8 theory does not distinguish between open, semi-open, and closed regions. Regions do not have to be internally connected, i.e. a regions may consist of different disconnected parts. It was shown by Duntsch [5, 4, 10, 8], that the composition of RCC8 is actually only a weak composition.

Weak composition ($\diamond$) of two relations S and T is defined as the strongest relation $R\varepsilon 2^A$ which contains the true composition SoT, or formally, $S \diamond T = R_i \varepsilon A | R_i \cap (SoT) \neq \phi$ [13].

Benett et al[1] call a weak composition table (entailed by Θ) extensional provided that the fact $CT(R,S) = \{T_1, \cdots, T_n\}$ always implies $\Theta \models \forall x \forall z \big[(T_1(x,z) \vee \cdots \vee T_n(x,z)) \leftrightarrow \exists y \big[R(x,y) \wedge S(y,z)\big]\big]$. Semantically speaking, this assures that , for any Θ model R and constants $a, c \in R$, the relational fact $T_i(a,c)$ also implies that there exists some constant $b \in R$ such that $R(a,b)$ and $S(b,c)$ holds, where T_i is a relation symbol taken from $CT(R,S)$. That is to say, Θ entails an extensional weak composition table if and only if each of its model is also an extensional model of this composition table.

Bennett [2] has pointed out that an extensional interpretation is not compatible with the RCC theory and had suggested the removal of the universal region u from the domain of possible referents of the region constants.

However, Li and Ying [9] proved that Bennett's conjecture is not valid and examined an RCC8 model comprising disks and regions with holes. They proved that no full extensional interpretation is possible. The extensionality of the table in the case of closed disks only has also been studied and proved [8]. Duntsch proposed RCC11 by considering the complement of a disk as a closed region (complemented closed disk algebra). Li and Ying [8] proved that the later algebra, whose domain contains only the closed disks and closures of their complements in the real plan is also an extensional model.

The above works so far have considered the extensionality of RCC8 for composition triads involving region with a hole. Because of the richness of the spatial domain, the results are not yet complete [10]. The approaches are based on visual reasoning with the triads and hence are not proven to be complete. They are also specific to the types of regions considered and could not therefore be generalized further to consider different models. Although exhaustive analysis was carried out for the cases considered, resulting in detailed labeling of the problem triads, explanations are lacking of when and under what conditions does the problem occur. The contribution of this work is two-fold. First, a method is proposed to identify conditions when extensionality is violated and secondly it is shown how extensional composition tables can derived by preserving the connectivity properties of space and its components.

3 Extensionality Problem of Composition Tables

Basic Notations:

For any $R,S,T \in R_8$, $\langle R,T,S\rangle$ denotes the fact that T is an entry in the cell specified by the ordered pair $\langle R,S\rangle$ in the RCC8 composition table, i.e. $T \in CT(R,S)$. As in [9] $\langle R,T,S\rangle$ is denoted a *composition triad*.

Our task is to verify, for each RCC model R and each triad $\langle R,T,S\rangle$ whether or not the following condition is true:

$$(\forall_{x,z} \in R)\big[\mathbf{T}_{x,z} \;\rightarrow\; (\exists y \in R)[\mathbf{R}(x,y) \wedge \mathbf{S}(y,z)]\big] \tag{1}$$

There are 178 possible triads in the RCC8 CT. If either R or S is the identity relation '$=$', then condition 1 is always true.

Regions in the RCC theory do not have to be open or closed or indeed internally connected and may consist of disconnected parts [3]. Three possible general configurations for regions can be distinguished in this theory, namely, closed disks (D), region with a hole (H) and a region with (at least two) disconnected parts (N), as shown in figure 1.

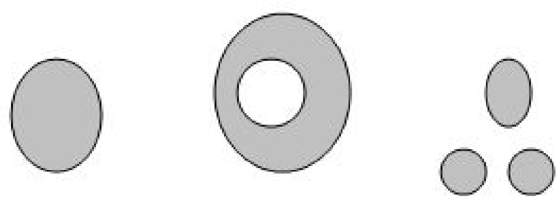


Fig. 1 Possible configurations of regions in topological space; closed disks, region with a hole and non-connected regions

Region triads are used to indicate the types of regions considered. Example of possible models include, $\langle D,H,N\rangle$, to indicate relations between a disk (D), region with a hole (H) and a region with non-connected parts (N), $\langle H,H,N\rangle$, $\langle N,N,N\rangle$, etc. In total, there are 16 possible triads.

The extensionality problem need to consider all possible permutations of the regions and hence verify condition 1 for all possible RCC models. The number of **composition triads** that needs to be considered is therefore 2848 (16 x 178).

Li and Ying [9] considered the region triads $\langle H,D,D\rangle$ (also equivalent to $\langle D,D,H\rangle$ as well as $\langle D,D,D\rangle$ in [10]. All possible relationships $T(x,z)$ satisfying these models were identified and then a visual search for region y was conducted to verify extensionality.

An annotated composition table is constructed where a superscript T^x is attached to each cell entry that leads to a non-extensional interpretation, as shown in table 1. All 178 cell entries were examined and the work concluded that an extensional model of the RCC8 composition table is only possible if the domain of possible regions is greatly restricted. In particular, regions with holes are disallowed. Also, that it would not be enough to ”remove the universal region of the possible referents of the region constants” and suggested by Bennett [2]. The work also suggests by means of an example that ”regions with two discrete components are possibly disallowed”, but did not provide any evidence for it.

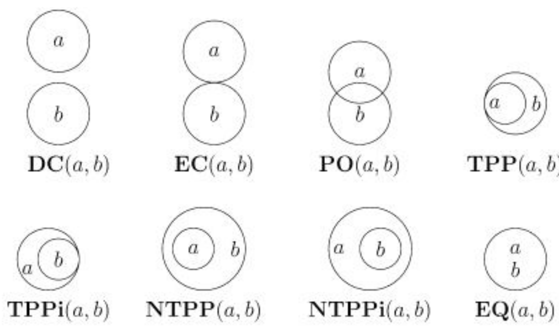


Fig. 2 Eight JEPD topological relations.

In what follows we use basic definitions for the interior, boundary and exterior of regions in a topological space. Let U be a topological space, $X \in U$ be a subset of U and $p \in U$ be a point in U.

- p is said to be an interior point of X if there is a neighborhood n of p contained in X. The set of all interior points of X is called the interior of X, denoted $i(X)$.
- p is said to be an exterior point of X if there is a neighborhood n of p that contains no point of X. The set of all exterior points of X is called the exterior of X, denoted $e(X)$.
- p is said to be a boundary point of X if every neighborhood n of p contains at least one point in X and one point not in X. The set of all boundary points of X is called the boundary of X, denoted $b(X)$.
- The closure of X, denoted $c(X)$, is the smallest closed set which contains X. The closure of a set is the union of its interior and its boundary.

Figure 3 shows the boundary, exterior and interior of the different regions considered in this work.

The question now is which of the 2484 triads imply non-extensional interpretation of the RCC8 CT.

$\circ$	DC	EC	PO	TPP	NTPP	TPPi	NTPPi
DC	$\top$	DC EC^x PO^x TPP^x NTPP	DC EC PO TPP NTPP	DC EC^x PO^x TPP NTPP	DC EC PO TPP NTPP	DC	DC
EC	DC EC^x PO^x TPPi^x NTPPi	DC EC^x PO^x = TPP TPPi	DC EC PO TPP NTPP	EC PO^x TPP^x NTPP	PO^x TPP^x NTPP	DC EC	DC
PO	DC EC PO TPPi NTPPi	DC EC PO TPPi NTPPi	$\top$	PO TPP NTPP	PO TPP NTPP	DC EC PO TPPi NTPPi	DC EC PO TPPi NTPPi
TPP	DC	DC EC	DC EC PO TPP NTPP	TPP NTPP	NTPP	DC = EC^x PO^x TPP	DC EC^x PO^x TPPi^x NTPPi
NTPP	DC	DC EC	DC EC PO TPP NTPP	NTPP	NTPP	DC EC^x PO^x TPP^x NTPP	$\top$
TPPi	DC EC^x PO^x TPPi^x	EC PO^x TPPi^x NTPPi	PO TPPi NTPPi	PO^x TPP TPPi =	PO^x TPP^x NTPP	TPPi	NTPPi
NTPPi	DC EC PO TPPi NTPPi	PO^x TPPi^x NTPPi	PO TPPi NTPPi	PO^x TPPi^x NTPPi	PO = TPP NTPP TPPi NTPPi	NTPPi	NTPPi

Table 1 Extensional Composition table for the RCC8 relations. R^x indicate composition results leading to non-extensional interpretation of the table .

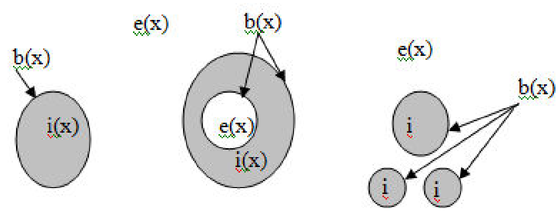


Fig. 3 Boundaries, interiors and exteriors of regions.

4 Connection and Extensionality

In any composition table involving three objects (x,y,z), for an entry in the table corresponding to objects x and z ($T(x,z)$) to be extensional requires the existence of another object y such that both relations $R(x,y)$ and $S(y,z)$ can be realised. If the resulting spatial configuration makes it impossible for a self-connected object to exist [1], then this entry is non-extensional.

[1] Recall that a region is called selfconnected if there is a path between any two points of the region that is completely contained in it.

In what follows, this connection property is used to study the extensionality properties of the RCC8 composition table and identify cases where extensionality is violated. If this property is explicitly modeled and considered in the reasoning formalism, the resulting composition table will always be extensional.

4.1 Extensionality of the Region Connection Calculus Composition Table

Three basic types of regions are normally considered in the literature, namely, a simple disk D, a region with a hole H and a disconnected region N. Sixteen possible region triads (D,H,N) need to be considered. One way to manage the study of the large number of possible relation triads in the composition table is to group them into two more general subsets; one with the set of containment relationship ($TPP, TPPi, NTPP, NTPPi$), denoted (containment composition) and the other group with the rest of possible relationships (DC, EC, PO), denoted (non-containment composition). In what follows, conditions for an extensional interpretation of the triad of relations, $R(x,y)$, $S(y,z)$ and $T(x,z)$ are identified.

Non-Containment Composition Figure 4 shows possible scenarios in the case of simple regions for objects x and z and their relationships with a common object y, where $T(x,z) \in \{DC, EC, PO\}$.

The condition for an extensional interpretation of the relation triad (R,T,S) can be informally expressed as follows. A connected line must exist outside both object x and z which also intersects the boundaries of both objects. One possible such line is shown as a dashed line is shown in figure 4.

Let δx and δz represent the boundary components of both objects x and z respectively and x° and z° are the exterior components. Let l be line object. The extensionality condition can be stated as follows: $l \cap \delta x \neq \phi \wedge l \cap \delta z \neq phi \wedge l \cap \{x^{\circ}, z^{\circ}\} \neq \phi$

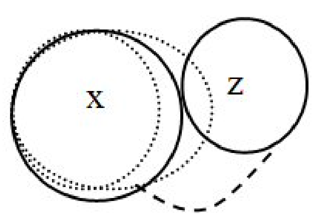


Fig. 4 x is touching, overlapping or disjoint from z. The connection constraint shown as a line joining the boundaries of the 2 objects.

Containment Composition A similar constraint can be defined for the case where $T(x,z) \in \{TPP, TPPi, NTPP, NTPPi\}$. The condition for an extensional interpretation of the relation triad (R,T,S) can be informally expressed as follows. A connected line must exist that joins the boundaries between both contained and container objects and this line must be completely embedded inside the container object. One possible such line is shown as a dashed line in figure 5.

This extensionality condition can be stated as follows: $l \cap \delta x \neq phi \wedge l \cap \delta z \neq \phi \wedge l \cap \{x^\circ, z^\circ\} \neq \phi$

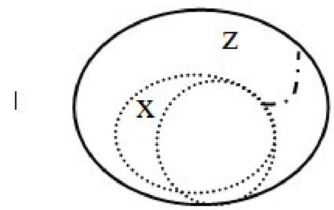


Fig. 5 x is contained in z. The connection constraint shown as a line embedded in z that joins the two boundaries of the 2 objects.

In what follows, we investigate the extensionality for the 16 possible combinations of region triads. We exclude the triad $\langle D,D,D\rangle$ which has already been proved.

Triads $\langle H,D,D\rangle$**,** $\langle D,N,H\rangle$**,** $\langle D,H,H\rangle$

This case has been studied in detail by Li [9] who enumerated all composition triads that will lead to non-extensional tables. 35 entries were identified, marked with a subscript (x) in table 1.

However, they failed to identify 4 possible triads based on the containment condition that will lead to non-extensionality condition. These triads are: $\langle DC,TPP,PO\rangle$, $\langle DC,TPPi,PO\rangle$, $\langle EC,TPP,PO\rangle$ and $\langle EC,TPPi,PO\rangle$.

Triads $\langle N,D,D\rangle$**,** $\langle D,H,N\rangle$

Figure 6(a) shows the relation $TPPi(N,D)$ that does not satisfy based on the containment condition. Examining its entry in the CT reveals that the following triads will therefore lead to non-extensional interpretations,

$\langle TPPi,TPPi,EC\rangle$, $\langle TPPi,TPPi,PO\rangle$, $\langle TPP,TPP,EC\rangle$, $\langle TPP,TPP,PO\rangle$.

Triads $\langle D,N,D\rangle$**,** $\langle D,H,D\rangle$

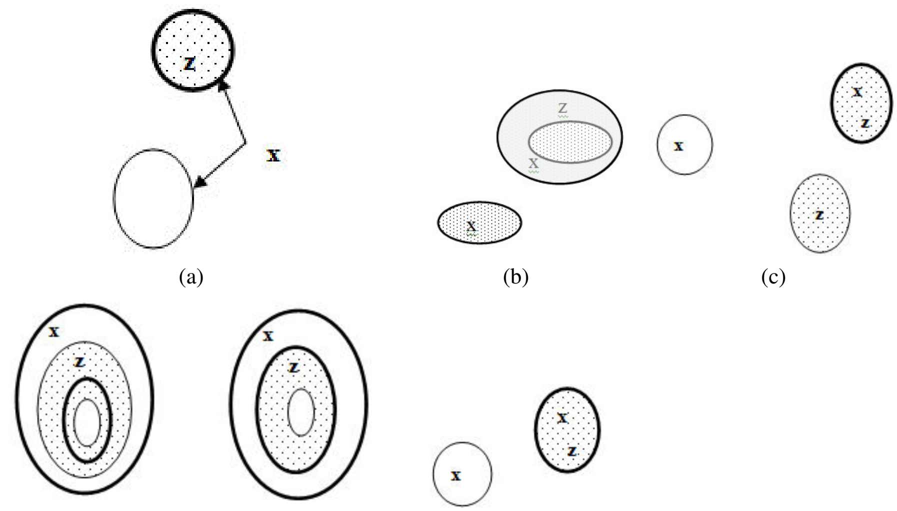
In this case, the relation $T(D,D)$ will always satisfy both conditions and so will always lead to extensional tables.

Triads $\langle H,N,N\rangle$**,** $\langle N,H,H\rangle$**,** $\langle N,D,H\rangle$

Figure 6(b) shows the relation $TPPi(H,N)$ that does not satisfy the containment condition. Examining its entry in the CT reveals that the following triads will therefore lead to non-extensional interpretations,

$\langle TPP,TPP,EC\rangle$, $\langle TPP!,TPP!,EC\rangle$, $\langle TPP,TPP,PO\rangle$, $\langle TPPi,TPPi,PO\rangle$.

Triad $\langle N,H,N\rangle$**,** $\langle N,D,N\rangle$



(a) (b) (c)

(d) (e)

Fig. 6 Cases of relations failing the extensionality condition. (a) Relation $TPPi(N,D)$ between X and Z. (b) Relation $EC(H,N)$ between X and Z. X is a non-connected region that completely overlaps Z. (c) Relation $PO(N,N)$ between X and Z. (d) Relation $PO(H,H)$ and $EC(H,H)$ between X and Z. (e) Relation $TPP(D,N)$ between X and Z.

The relation PO(N,N), as shown in figure 6(c) leads to non-extensionality based on non-containment condition with the following triads: $\langle EC,PO,TPP\rangle$, $\langle PO,PO,TPP\rangle$ and $\langle TPP,PO,PO\rangle$

Triad $\langle H,N,H\rangle$ The composition table is extensional because the third object is not connected.

Triad $\langle H,D,H\rangle$

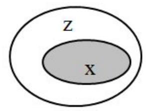
Here, the triads: $\langle EC,PO,EC\rangle$ and $\langle EC,EC,EC\rangle$ are problematic as shown in figure 6(d), due to violating the non-containment condition.

Triad $\langle D,N,N\rangle$

Here, the triads: $\langle TPP,TPP,EC\rangle$ and $\langle TPP,TPP,PO\rangle$ and $\langle PO,TPPi,EC)\rangle$ are problematic as shown in figure 6(e), as the containment condition can not be satisfied.

5 Verifying the Extensionality of the Composition Table

Consider the region triad (D,D,H) and the relation triad (EC,EC,EC). This triad has been shown to be non-extensional in the case of the relation in figure 7(III). In this case, the disk x completely coincides with the hole in object z. This case is considered to be non-extensional as no object y can be found such that $EC(x,y)$ and $EC(y,z)$.



(a)

	z_{01}	z_1	z_2	z_3	z_{02}
x_0	1	1	1	0	0
x_1	0	0	0	1	0
x_2	0	0	0	0	1

(b)

Fig. 7 (a) $EC(x,z)$, where x is a disk and z is a region with a hole. (b) Corresponding intersection matrix.

Here, we examine this composition table entry more closely. We identify all possible scenarios for the composition and use a representation and reasoning approach that consider the connectivity of the space and its components to derive the resulting relationships between x and z. It is shown how the method results only in extensional relationships and hence demonstrates that the general treatment of objects as in the case of RCC8 is limited and that more specific representation of object complexity are needed.

In figure 8 the object and space components are labeled to indicate their different interior, boundaries and exteriors. In figure 9(III) a diagrammatical representation of the spatial relations for $R(x,y)$ and $S(y,z)$ are shown. Intersection matrices associated with the relationships are also shown to depict the intersection between their individual components. Three possible EC relationships can exist between y and z.

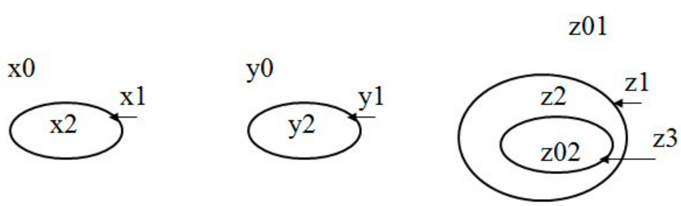
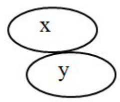
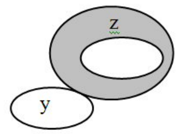


Fig. 8 Object and space components for regions x (disk), y (disk) and z (region with hole).

The non-extensional relationship between x and z is shown in figure 7 along with its corresponding intersection matrix.

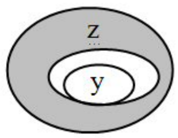


	x_0	x_1	x_2
y_0	1	1	1
y_1	1	0	0
y_2	1	0	0



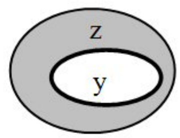
	z_{01}	z_1	z_2	z_3	z_{02}
y_0	1	1	1	1	1
y_1	1	1	0	0	0
y_2	1	0	0	0	0

Case I



	z_{01}	z_1	z_2	z_3	z_{02}
y_0	1	1	1	1	1
y_1	0	0	0	1	1
y_2	0	0	0	0	1

Case II



	z_{01}	z_1	z_2	z_3	z_{02}
y_0	1	1	1	0	0
y_1	0	0	0	1	0
y_2	0	0	0	0	1

Case III

Fig. 9 $EC(x,y)$ and three different possible relationships $EC(y,z)$.

The composition $R(x,y) \circ S(y,z) \to T(x,z)$ is derived by composing their corresponding matrices and propagating the empty and non-empty intersections using the formalism defined in [7, 6]. Using the representation scheme of object and space as in figure 9, two reasoning rules are used to propagate empty and non-empty intersections between object components and the result is encoded in an intersection matrix. The rules are as follows.

Rule 1: Propagation of Non-Empty Intersections

Let $x' = \{x_1, x_2, \cdots, x_{m'}\}$ be a subset of the set of components of space X whose total number of components is m and $m' \leq m$; $x' \subseteq X$. Let $z' = \{z_1, z_2, \cdots, z_{n'}\}$ be a subset of the set of components of space Z whose total number of components is n and $n' \leq n$; $z' \subseteq Z$. If y_j is a component of space Y, the following is a governing rule of interaction for the three spaces X, Y and Z.

$$
\begin{aligned}
(x' \sqsupseteq y_j) \;\; \wedge \;\; & (y_j \sqsubseteq z') \\
\to \;\; & (x' \cap z' \neq \phi) \\
\equiv \;\; & (x_1 \cap z_1 \neq \phi \vee \cdots \vee x_1 \cap z_{n'} \neq \phi) \\
& \wedge (x_2 \cap z_1 \neq \phi \vee \cdots \vee x_2 \cap z_{n'} \neq \phi) \\
& \wedge \cdots \\
& \wedge (x_{m'} \cap z_1 \neq \phi \vee \cdots \vee x_{m'} \cap z_{n'} \neq \phi)
\end{aligned}
$$

The above rule states that if the component y_j in space Y has a positive intersection with every component from the sets x' and z', then each component of the set x' must intersect with at least one component of the set z' and vice versa.

The constraint $x_i \cap z_1 \neq \phi \;\vee\; x_i \cap z_2 \neq \phi \cdots \;\vee\; x_i \cap z_{n'} \neq \phi$ can be expressed in the intersection matrix by a label, for example the label a_r (r = 1 or 2) in the following matrix indicates $x_1 \cap (z_2 \cup z_4) \neq \phi$ (x_1 has a positive intersection with z_2, or with z_4 or with both). A $-$ in the matrix indicates that the intersection is either positive or negative.

	z_1	z_2	z_3	z_4	$\cdots$	z_n
x_1	$-$	a_1	$-$	a_2	$-$	$-$

Rule 1 represents the propagation of non-empty intersections of components in space. A different version of the rule for the propagation of empty intersections can be stated as follows.

Rule 2: Propagation of Empty Intersections

Let $z' = \{z_1, z_2, \cdots, z_{n'}\}$ be a subset of the set of components of space Z whose total number of components is n and $n' < n$; $z' \subset Z$. Let $y' = \{y_1, y_2, \cdots, y_{l'}\}$ be a subset of the set of components of space Y whose total number of components is l and $l' < l$; $y' \subset Y$. Let x_i be a component of the space X. Then the following is a governing rule for the spaces X, Y and Z.

$$(x_i \sqsubseteq y') \;\wedge\; (y' \sqsubseteq z') \\ \rightarrow (x_i \cap (Z - z_1 - z_2 \cdots - z_{n'}) = \phi)$$

Remark: if $n' = n$, i.e. x_i may intersect with every element in Z, or if $m' = m$, i.e. z_k may intersect with every element in X, or if $l' = l$, i.e. x_i (or z_k) may intersect with every element in Y, then no empty intersections can be propagated for x_i or z_k. Rules 1 and 2 are the two general rules for propagating empty and non-empty intersections of components of spaces.

Note that in both rules the intermediate object (y) and its space components plays the main role in the propagation of intersections. The first rule is applied a number of times equal to the number of components of the space of the intermediate object. Hence, the composition of spatial relations becomes a tractable problem which can be performed in a defined limited number of steps.

The result intersection matrices for the three scenarios are shown below. In every case, the matrix is shown not to propagate the non-extensional matrix in figure 7.

Case I:

The result matrix for case I is as follows.

	z_{01}	z_1	z_2	z_3	z_{02}
x_0	1	b	?	?	?
x_1	a	a b	?	?	?
x_2	?	?	?	?	?

In the matrix above, ? is used to denote either 0 or 1. Letters are used to signify related constraints. So, entries labeled a denotes that at least one of the entries x_1, z_0 and x_1, z_1 should be 1.

As can be seen, the matrix above holds the constraint, $x_1 \cap (z_{01} \cup z_1) \neq \phi$. This constraint contradicts with the non-extensional matrix, where the constraint is

$x_1 \cap (z_{01} \cup z_1) = \phi$. Hence, the composition will not propagate the non-extensional relation.

Case II:

In a similar fashion to Case I, the result of composition for case II is as follows.

	z_{01}	z_1	z_2	z_3	z_{02}
x_0	?	?	?	b ?	1
x_1	?	?	?	a b	a
x_2	?	?	?	?	?

Here, the contradictory constraint is $x_0 \cap z_{02} \neq \phi$ and hence, this composition will not propagate non-extensional relations.

Case III:

Similarly for case III, the result matrix is as follows.

	z_{01}	z_1	z_2	z_3	z_{02}
x_0	?	?	?	1	1
x_1	?	?	?	1	0
x_2	?	?	?	0	0

Here, the contradictory constraints are $x_0 \cap z_3 \neq \phi$ and $x_0 \cap z_{02} \neq \phi$.

It all three cases it was shown how the representation and reasoning methods result only in extensional relationships.

6 Conclusions

This paper addresses the question of extensionality of RCC8 composition table. Sixteen possible region triads have been identified that needs to be studied for this problem. Two of which have been addressed exhaustively in earlier works, but their methods could not be easily extended or generalized. We demonstrate the value of considering the connectivity of object and space components in deriving extensional composition tables. Two general conditions for extensionality are proposed and used to exhaustively test the exhaustively test the different types of regions in the RCC8 composition table. It is observed that the table will be extensional only in some cases ($\langle D,D,D \rangle$, $\langle D,H,D \rangle$, $\langle D,N,D \rangle$ and $\langle H,N,H \rangle$.). The result is important as ignoring the types of regions in the application of the spatial logics may lead to the propagation of non-valid reasoning results.

The results of this paper complements earlier work on regions with holes as well as clarifies the fact that extensional interpretation is not violated automatically by the existence of a non-disk object, but in fact will depend on the order in the composition triad. Further extension to this work is sought to investigate other existing composition tables and models of spatial calculi.

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