# An Intersection-Based Formalism for Representing Orientation Relations in a Geographic Database

Alia I. Abdelmoty † and Baher A. El-Geresy ‡
† Dept. of Computer Studies, University of Glamorgan,
Pontypridd, Mid Glamorgan, CF37 1DL, Wales, U.K.
‡ Dept. of Mechanical Engineering, Heriot-Watt University
Riccarton, Edinburgh, EH14 4AS, Scotland, U.K.

## Abstract

This paper presents a representation formalism for the definition of general qualitative orientation relationships, an important category of relationships in a geographic database (GDB), and takes into account the effect of the shape, size and proximity of the objects involved. The formalism follows an intersection-based approach of representation which is characterised by a sound mathematical basis and applicability in a geographic database context. Representation formalisms for qualitative spatial relations are the basis for qualitative spatial reasoning used to infer spatial relationships which are not stored explicitly in the database, to answer spatial queries given partial spatial knowledge and to maintain the consistency of the GDB.

## 1 Introduction

A major challenge in the development of geographic databases is the representation and derivation of spatial relationships. It is generally agreed that it is neither practical nor efficient to store the substantial number of different types of spatial relationships that can exist in the geographic space. Qualitative reasoning mechanisms has been proposed for the automatic derivation of spatial relations which complements computationally expensive computational geometry. Qualitative reasoning is based on the manipulation of qualitative spatial relationships. Representation formalisms for modelling qualitative relations are being studied for topological and orientation relations and extensions to query languages have been developed to incorporate some of these.

Spatial properties of the objects considered, in particular, their shape and size, as well as their relative proximity play an important role in the determination of their relative relationship. In this paper, the effect of these factors on the representation of orientation relationships is studied and a representation formalism for the definition of this type of relationships is presented. The formalism developed also handles different types of orientation relations (depending on the frame of reference adopted). For example, in determining the relationship of a river object with respect to a house object located on its bank, the relation can be either, the river is in-front of the house, taking the point of view of the house, or, the river is to the left of the house, taking the point of view of an observation point to the left of the house, or, the house is on the west bank of the river, taking a global reference frame.

The formalism developed is an extension to the intersection-based approach developed originally by Egenhofer [EH90, Ege89a] for representing topological relations, and carries its sound mathematical basis and suitability for implementation in a GDB.

The paper is structured as follows. Section 2 outlines several properties which representation formalisms for qualitative spatial relations aims to achieve and describes briefly the approach which is extended in this paper. Section 3 introduces a qualitative frame of reference which is used for classification of the different types of spatial relations and for studying their interrelationships. Section 4 presents the extension to the intersection-based approach to the representation of general orientation relationships. Section 5 shows how the formalism can be adapted to take several affecting factors into account. In section 6 a comparison of the proposed approach with others in the literature is given and some conclusions are presented in section 7.

## 2 Formalisms for Representing Spatial Relations

A representation formalism is basically a set of constraints specified to define a set of spatial relationships. Two approaches can be recognized for developing such formalisms, as shown in figure 1, viz, a *constraint-driven* approach which starts by defining a general set of constraints based on which a set of relationships can be defined, and a *relation-driven* approach which starts by recognizing the set of relationships (intuitively), and then identifying the set of constraints necessary for their unique definition.

An example of the constraint-driven approach to representing topological relationships is that developed by Egenhofer [Ege89a], where objects and their embedding space are represented in terms of their components, namely, interior $A^\circ$, boundary $\delta A$ and exterior $A^{-1}$. The combinatorial intersection of these components represent the set of constraints which collectively define the relationships. An intersection-matrix representing the intersection of components is used as the model in this approach, and is,

$$\begin{bmatrix} \delta A \cap \delta B & \delta A \cap B^\circ & \delta A \cap B^- \\ A^\circ \cap \delta B & A^\circ \cap B^\circ & A^\circ \cap B^- \\ A^- \cap \delta B & A^- \cap B^\circ & A^- \cap B^- \end{bmatrix}$$

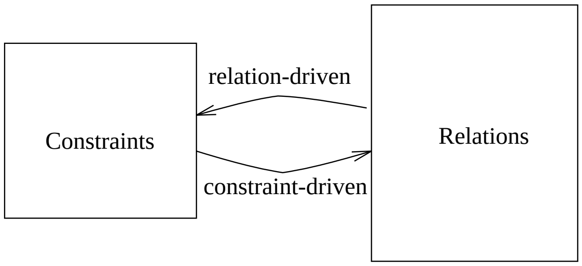


Figure 1: Different approaches to the representation of qualitative spatial relations.

An example of the relation-driven approach to representing topological relationships is that developed by Randell et. al [RCC92], where a set of axioms for defining every needed relationship have to be devised. For example, the definition of the `overlap` relationship between two simple regions is : $Overlap(x, y) \leftarrow Part(z, x) \wedge Part(z, y)$ and a set of constraints are used for defining the relationship $Part(x, y)$.

Any formalism for the representation of qualitative spatial relationships can be judged by its ability to satisfy the following criteria.

1. Completeness: where the set of constraints used define a complete set of relations in the domain studied, i.e. there exist a subset of constraints for every possible relation in the domain.

2. Soundness: where the combination of the set of constraints used can only produce a feasible or physically correct set of relations.

3. Uniqueness of representation: where the set of constraints used can uniquely distinguish every possible qualitative relation in the domain. Different levels of granularity of spatial relations can be required to be distinguished. For example, on one level of granularity cardinal directions can be north, east, south and west, whereas using a finer granularity, relations such as north-east and south-west might be needed to be distinguished.

4. Generality: which is the ability of the formalism to represent different types of spatial relations between different shapes of spatial objects.

In figure 2 an illustration of the first three criteria is shown. $C_c$ are the set of constraints necessary for the definition of a complete set of relations $R_c$. $R_{s'}$ is a set of sound relations and $C_{s'}$ are their corresponding constraints. $C_s$ are the set of constraints needed for defining the set of complete and sound relations $R_s$ and $C_u$ are the set of constraints needed for the definition of a unique set of relations $R_u$. As shown in 2(a), satisfying the completeness criteria alone does not guarantee that the resulting set of relations are all feasible or physically correct and similarly in figure 2(b), a representation formalism which aims to satisfy the soundness criteria alone does not necessarily guarantee the completeness criteria. In figure 2(c) the set of constraints $C_u$ needed for satisfying the uniqueness criteria are always larger that the sets $C_c$ and $C_s$.

The representation formalism developed by [Ege89a] was shown to satisfy the completeness criteria [EF91, CDO93]. However, extra constraints had to be defined to satisfy soundness [Ege89b, JB94, iVM94]. To achieve a certain level of uniqueness, more constraints were used, such as recording the dimension of the resulting intersection [FE92, CDO93] and the number of occurances of particular intersections [FE92]. The basic approach was used for representing topological relations between two simple regions. However, it has been extended for representing lines [Ege93], regions with holes [Ege94], and raster regions [ES93]. Abdelmoty & Williams [AW94] extended the approach for the representation of cardinal direction relations and for the representation of flow direction relations between linear dynamic geographic abstract data types [APWAF94]. In this work this approach is used for the representation of general orientation relations which are generalization of cardinal direction relations.

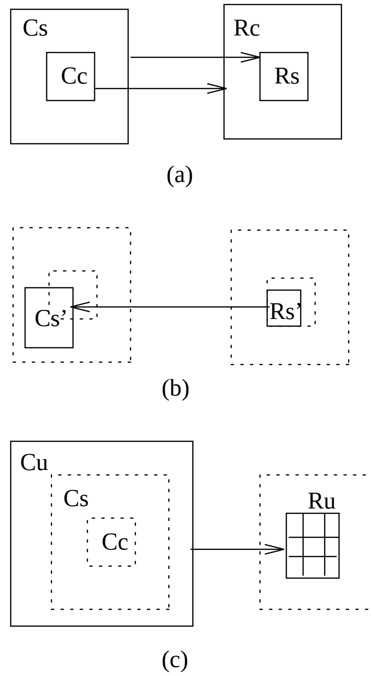


Figure 2: Mapping between constraints and relations for the criteria of (a) completeness, (b) soundness and (c) uniqueness.

## 3 The Qualitative Frame of Reference

Many classifications exist for qualitative spatial relations. The most common classification is between topological, order, and metric relationships [EH90]. Other classifications were suggested, for example, Freeman [Fre75] distinguishes between two types, those dependent on relative properties (darker, larger) and those dependent on relative position (near, above) and McDermott & Davis [MD84] distinguish between three types based on relative position, relative orientation and relative scale (size). Hernandez [Her94] recognizes the topological and orientation relations as the two factors which can be considered independently to determine the relative position of the object in 2D space.

To study the representation and definition of spatial relations, a qualitative frame of reference is proposed which is analogous to the quantitative frame of reference used to define the position of objects. This qualitative frame will serve as a classification tool with which the interrelationships between the different types of spatial relations can be studied.

In a quantitative frame of reference an 'origin' or a point of reference is used and the variations or degrees of freedom of another point in space with respect to the 'origin' are represented as variables on axes. The values which these variables takes are the 'coordinates'. For example in a 3D space an object location is fully described by values on the $x$, $y$, and $z$ axes or by values on the $r$, $\theta$ and $\gamma$ axes. The main features of such coordinates are as follows.

- The collective values of the coordinates fully describe the relations between the origin and the referenced object.
- Variations along the different axes represent different degrees of freedom of the referenced object with respect to the 'origin'. These variations are independent. For example, values on the $x$ axis can vary while values on the $y$ and $z$ axes remain constant.
- Possible variations of coordinates along each axis are continuous, ordered and non-overlapping.

Bearing these requirements in mind, a qualitative frame of reference can be established for spatial relations of an object with respect to another, where one of the objects represents the 'origin' with respect to which the other object is referenced. An object in space possess three degrees of freedom which determine its spatial relationship with other objects. These are transition, rotation and scaling (enlargements or shrinking). Accordingly, three axes of variation can be established as shown in figure 3, namely,

1. Interaction-proximity axis: over which the variation represents relationships resulting from the relative transition of objects.
2. Orientation axis: over which the variation represents relationships resulting from the relative rotation of objects.
3. Size axis: where scaling variations are represented.

Checking the properties of this frame of reference against the set of requirements of the quantitative frame of reference reveals the following.

1. The different types of spatial relations recognized can be represented along those axes.
2. Variations along each coordinate are continuous, ordered and non-overlapping. For example, variation on the Interaction-Proximity axes yields relations such as far, close, touch, overlap, equal.
3. Inter-dependencies of the axes can be recognized as follow,
   (a) Relationships along the Size axis is independent of the other two axes.
   (b) Relationships along the Interaction-Proximity axis can be dependent on the size of the objects involved in the case where the objects are in close proximity. For example, when objects are very close, changing the size of the objects can transform a relationship of `disjoint` into `overlap` or the relationship of `equal` to `contain` or `inside`.
   (c) Relationships on the Orientation axis can be affected by both size and proximity of the objects involved. An object `east` of another can become also `north` or `south` of it, if it increases in size or gets closer to the other object.

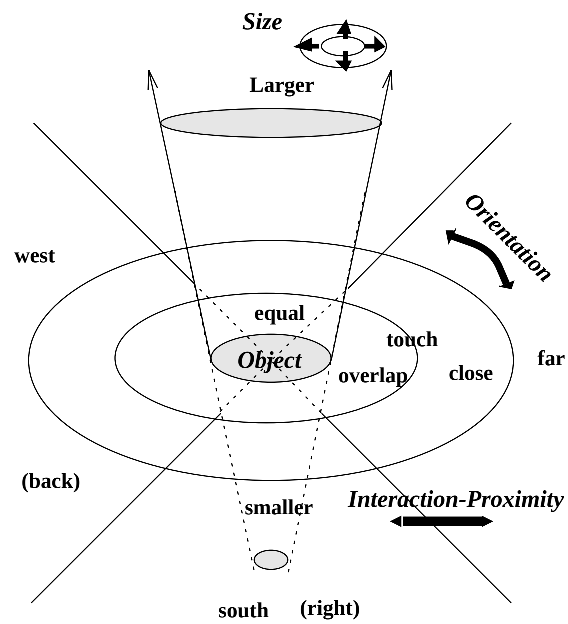


Figure 3: Qualitative frame of reference for spatial relationships.

This analysis reveals that the closer the objects are, the more stronger the dependency between the different types of relations. This stresses the need for careful investigation of the effect of size and proximity of objects on relationships on the Orientation axis in particular. Note that the above frame of reference is independent of the dimension of the embedding space. Examples of relations on the Orientation axis in a 2D space are, `east`, `west`, `in-front` and `back` and in a 3D space are, `above` and `below`. Another observation is that spatial relations such as `between` is not represented using this frame of reference since it involves two or more reference objects and not one as assumed here.

## 4 Representing Relations on the Orientation Axis

Different types of orientation relations can be recognized on the Orientation axis, depending on the frame of reference used. Retz-Schmidt [RS88] identified three frames of reference namely,

**Intrinsic:** when the orientation is determined by some inherent property of the reference object (front of the house).

**Extrinsic:** when the orientation is determined by a fixed external frame of reference (east, west, north, south).

However, there is a relation between the above two frames of reference in case of stationary objects (the house front faces north).

**Deictic:** when the orientation is imposed by an observer or point of view object inside the scene itself.

Several approaches have been proposed in the literature for the representation of different types of orientation relations. Mukerjee and Joe [MJ90, MJ89] investigated the intrinsic orientation relation using the minimum bounding rectangle (MBR) of objects and a rectangular division of the space. Freksa [Fre92] used a similar approach as [MJ90] for representing deictic type of orientation relations using point abstractions of objects. Hernandez [Her94] proposed a conical representation to take into account the effect of proximity of the reference object at different levels of granularity. An earlier study was carried out by Peuquet & C-Xiang [PCX87] on the effect of shape and proximity on the extrinsic orientation relations (cardinal direction) using a conical model of the space. Frank [Fra92] proposed a rectangular division of cardinal (extrinsic) relations and demonstrated its advantages in reasoning with orientation relations. Papadias & Sellis [PS93] used a similar division for cardinal directions to that in [Fra92] based on symbolic spatial indexes. Also, Cui et al [CCR93] used a theory based on first-order logic to represent extrinsic and deictic orientation relations. Jungert [Jun94] presented a generalised symbolic projection method (slope projection) for the definition of extrinsic orientation relations. Abdelmoty & Williams [AW94] described an intersection-based approach (constraint-driven) for the representation of the extrinsic orientation (cardinal direction) relations.

The use of intrinsic and deictic orientation can be feasible and useful in the geographic space where objects are stationary. For example, buildings have fronts and backs, river banks can be referenced according to the direction of flow of the water as left and right. Thus relations such as, "the car is in-front of the house", can be used.

However, relations in the intrinsic frame of reference are not order relations while relations in the extrinsic and single observer deictic frames are order relations. For example, the relationship "the car is `in-front` of the house", does not imply that "the house is `behind` the car", nor does it imply any other relation. Thus, it cannot be used in the reasoning process, unless another relation is defined for example, "the house is to the left of the car". This problem will be addressed using the formalism developed in the rest of this section.

### 4.1 The Formalism

Consider figure 4 where two objects $x$ and $y$ are shown and an intrinsic frame of reference is used. A conical model of the space is used with point abstractions of the objects. The following relationships exist:

$$y\ [in-front\ of]\ x$$
$$x\ [left\ of]\ y$$

Let $x_F$ denote the semi-infinite area defining `in-frontof` $x$ as shown in figure 4. Thus the relation $y\ [in-front]\ x$ implies the relation $y\ inside\ x_F$. The boundaries of the area $x_F$ will intersect with the boundaries of one or more of the semi-infinite areas defining the exterior of object $y$ resulting in one or more finite areas (out of four at this level of granularity).

Only one of those finite areas, namely, the one which has both objects as part of its boundary (shown shaded in figure 4), can determine the orientation relationship between the two objects. This fact can be proved since $y\ inside\ x_F$ and $x\ inside\ y_L$ imply that the two areas $x_F$ and $y_L$ must intersect in a finite area with objects $x$ and $y$ as part of its boundaries.

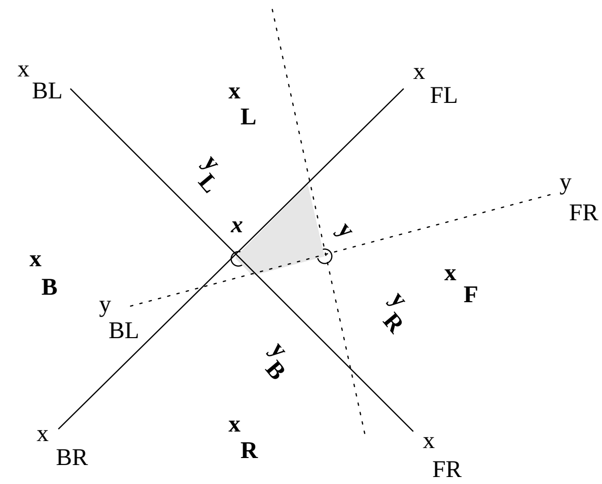


Figure 4: Objects in an intrinsic frame of reference. The shaded area is used to determine the relative orientation of the objects.

The intersection of the semi-infinite areas can be substituted by the intersection of their boundaries represented by directional lines separating them, denoted, $x_{FL}$ for `front-left`, $y_{BR}$ for `back-right`, etc. An intersection-matrix representing the combinatorial intersection of these orientation lines as well as the body (represented as a point in this case) can be used to determine the orientation relationship between the two objects as follows.

$R(x, y) =$

| | $x$ | $x_{FL}$ | $x_{FR}$ | $x_{BL}$ | $x_{BR}$ |
|---|---|---|---|---|---|
| $y$ | | | | | |
| $y_{FL}$ | | | | | |
| $y_{FR}$ | | | | | |
| $y_{BL}$ | | | | | |
| $y_{BR}$ | | | | | |

Note that using this representation mechanism both the relations $x[R_1]y$ and $y[R_2]x$ can be determined which solves the problem stated in the beginning of this section. From the above analysis, the orientation relations can be derived using the following general rules.

**Rule 1** *An object $y$ is said to be in the direction $d_j$ of another object $x$, $x[d_j]y$, if the following set of intersections are true in the orientation intersection matrix,*

*1.*

$$x_{ij} \cap y_{mq} = 1$$
$$x_{jk} \cap y_{lm} = 1$$
$$x_{ij} \cap y_{lm} = 0$$
$$x_{jk} \cap y_{mq} = 0$$

*In this case, $x$ is said to be in the direction $d_m$ of $y$, $x[d_m]y$ and $y[d_j]x$, as shown in figure 5(a), or,*

*2.*

$$x \cap y_{lm} = 1$$
$$x_{ij} \cap y_{mq} = 1$$
$$x_{jk} \cap y_{ln} = 1$$

*In this case, $x$ is said to be in the direction $d_{lm}$ of $y$, $x[d_{lm}]y$, as shown in figure 5(b).*

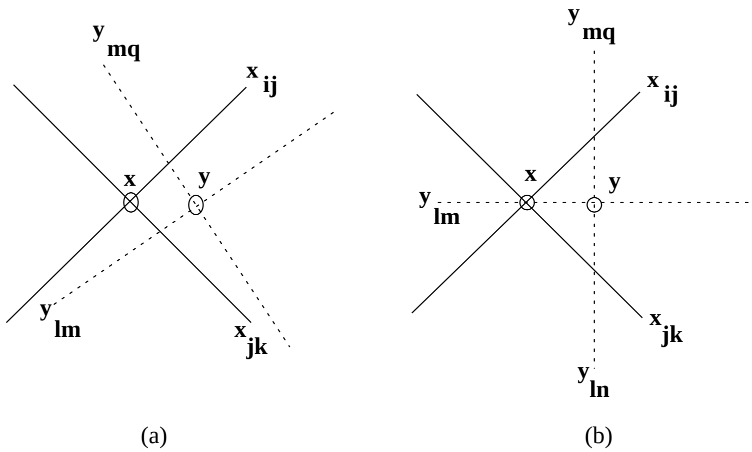


Figure 5: Defining the orientation relation using the intersection approach.

The above rules are applicable for any level of granularity of orientation relations and for any frame of reference; deictic, extrinsic or intrinsic, Special cases of the above general rules apply for the extrinsic and deictic frames of reference, where the orientation lines of the two objects are parallel and the resulting orientation relation is an order relation. (In the case of the deictic frame of reference, we assume the two objects are close enough to make their orientation lines parallel). In these cases the following rule applies,

**Rule 2** *(In extrinsic or deictic frame of reference)*
*An object $y$ is said to be in the direction $[d_j]$ (or $[d_{ij}]$) of another object $x$ (i.e., $y[d_j]x$ or $xd_{\bar{j}}y$ where $d_{\bar{j}}$ is the converse relation of $[d_j]$) if the following set of intersections are true in the orientation intersection matrix,*

1. *if two directional lines of two bodies intersect such that, $x_{ij} \cap y_{k\bar{j}} = 1$, where $k = i; j$, or,*
2. *if two directional lines of two bodies intersect such that, $x_{ij} \cap y_{\bar{i}\bar{j}} = 1$, then $y[d_{ij}]x$ and $x[d_{\bar{i}\bar{j}}]y$.*

The above intersection relations is enough to describe the orientation relation between the two objects, $x[R_1]y$ and $y[R_2]x$, in all the frames of reference. Note that in the case of the intrinsic frame of reference the resulting relations are not order relations.

## 5 Orientation-Interaction Plane

Spatial queries, might involve the derivation of orientation relations between two objects which are not disjoint. Objects, in this case, can either overlap or contain each other, and the intersection of the first element of the orientation matrix will be non-empty.

The size of the objects cannot be ignored in those cases, and can be approximated using the minimum bounding rectangle (MBR). The sides of the MBR has therefore to be accounted for in the intersection matrix, where they form boundaries of the semi-infinite areas forming the exterior of the objects involved. The intersection matrix in this case is as follows,

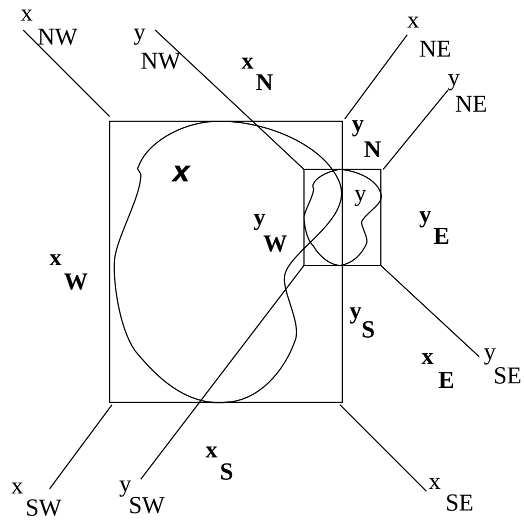


Figure 6: Orientation relations between overlapping objects.

| | $x$ | $x_{NE}$ | $x_{NW}$ | $x_{SE}$ | $x_{SW}$ |
|---|---|---|---|---|---|
| $y$ | 1 | ? | ? | ? | ? |
| $y_{NE}$ | ? | ? | ? | ? | ? |
| $y_{NW}$ | ? | ? | ? | ? | ? |
| $y_{SE}$ | ? | ? | ? | ? | ? |
| $y_{SW}$ | ? | ? | ? | ? | ? |

### 5.1 Overlapping Orientation

Figure 6 illustrates a situation where the cardinal direction between objects $x$ and $y$ is required. Let $x_E$, $x_N$, $x_S$, $x_W$ and $y_E$, $y_N$, $y_S$, $y_W$ be the sides of the MBR of objects $x$ and $y$ respectively.

The rule for the definition of the orientation relation in this case is,

**Rule 3** *An object $y$ is said to be in the direction $[d_j]$ of another object $x$ if $(x_j; x_{ji}; x_{j\bar{i}}) \cap (y_j; y_i; y_{\bar{i}}; y_{\bar{j}}; y_{\bar{j}i}; y_{\bar{j}\bar{i}}) = 1$*

Applying this general rule to the case in figure 6, we can derive the relation $y[East]x$, since, $(y_N, y_S) \cap x_E = 1$, and the relation $x[West]y$, since, $(y_{SW}, y_{NW}) \cap x_W = 1$, where $x_E$ denotes one of the lines of the MBR of $x$ and $y_{NW}$ is an orientation line of object $y$, and so on.

### 5.2 Containment Orientation

In many GIS applications the orientation of a certain object with reference to an object which contains it is needed. Relations such as, Edinburgh is in the north of Britain, while Cardiff is in the west, or, Egypt is in the north of Africa, and Singapore is in south-east Asia are obvious examples. Orientation relations in this case are distinguished by the fact that the orientation areas are finite and represented by the interior of the reference object and not by its exterior (this relation can be called the directional position of the object).

While the conical model is acceptable when considering usual orientation relations, (dividing the 'infinite' exterior of the objects), it is probably not as obvious when defining orientation acceptance areas in a small finite space. In this case, a more cognitively accepted division could be the rectangular division (see Mark [Mar92]), as shown in figure 7.

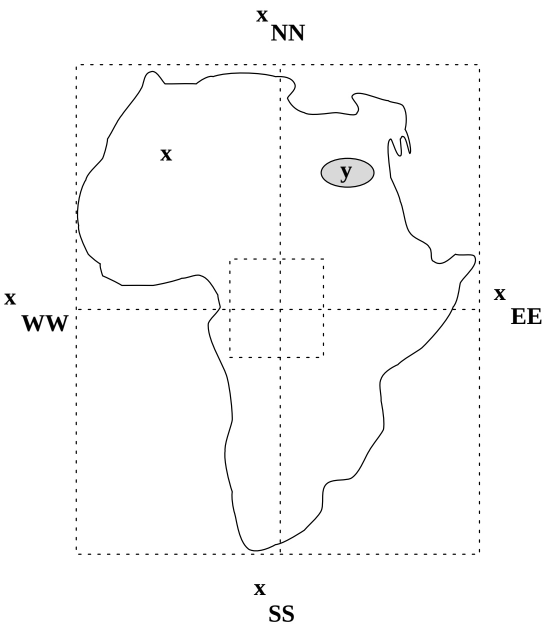


Figure 7: Orientation relations when one object is inside the other.

An area in the middle of the reference object can be considered as `central` or `middle` of the object for example, city centre, central Africa, etc.

The shape of the reference object can be used to define the taxonomy (or level of granularity) of the orientation relations. For example, if the north-south extension of the object is much larger than its east-west extension, a coarse granularity of only one level could be acceptable (north and south only, with a strip in the middle to represent the central part). The situation will be reversed if the east-west extension is larger than the north-south. In figure 7 a more general case is represented where four divisions are used. Note that only the orientation relation of the smaller object with respect to the larger one is meaningful in this case.

The rule for defining the orientation relation in the case shown in figure 7 is $y[North]x$ if $x_{NN} \cap (y_{EE}; y_{WW}; y_{SS}) = 1$[1]. More generally, the rule can be expressed as follows,

**Rule 4** *An object $y$ is said to be in the directional position $[d_j]$ of another object $x$ containing it, if $(x_{jj} \cap (y_{ii}; y_{\bar{i}\bar{i}}; y_{\bar{j}\bar{j}}) = 1$.*

The relation of $y[Center]x$ can only be decided if the order of magnitude of the length of intersected directional lines of the referenced object is small compared to the length of directional lines of the reference object.

## 6 Generality of the Formalism

Table 1 presents a comparative summary of the present approach with the other approaches proposed in the literature. The comparison is carried out from the following aspects:

1. The orientation frame of reference which is covered by the approach, whether extrinsic, intrinsic or deictic.
2. The specific space division adapted in the formalism whether conical or rectangular (where some acceptance areas are defined by parallel lines).
3. The constraints used in the definition of the orientation relation is either using the `inside` relationship, where the object lies `inside` an acceptance area, or using an `order` relation ($<, =, >$), which is the case in the projection approaches, or through an `intersection` relation, which is the approach used in this paper. Since the extrinsic and deictic relations are converse relations while the intrinsic relations are not, in the case of extrinsic and deictic relations, determining the relation of one object relative to the other implies the opposite relation, for example, `east(a,b)` implies `west(b,a)`, In the case of the intrinsic relations, other approaches which use constraints defined through the `inside` relation can only represent the relation of the prime object with respect to the reference one and not the opposite. However, our approach can define both relations using the same set of intersections.
4. Types of objects whether point abstractions or extended objects.
5. The validity of the representation formalism for representing the orientation relations when objects considered are in different relations on the interaction-proximity axis.

[1] An intersection result of 1 is used to denote a non-empty intersection.

## 7 Summary and Conclusions

A formalism for the representation of orientation relations was presented which is based on the intersection-based approach of representation and which takes into account the effect of the size, shape and proximity of the considered objects. Some conclusions can be drawn as follows,

- Four criteria were identified against which representation formalism can be measured, namely, soundness, completeness, uniqueness of representation and generality.
- Three axes for a qualitative frame of reference for spatial relations were identified, namely, interaction-proximity, orientation, and size.
- Orientation relations are not completely independent of the other two relations, in particular in the case where the objects are in close proximity.
- An intersection-based formalism for the representation of orientation in the intrinsic frame of reference was introduced based on the intersection of orientation direction lines.
- Rules for representing the orientation relations when objects are intersecting were represented to account for the effect of proximity, shape and size of the objects involved.

The intersection-based approach for representing orientation relations was shown to be simple, in terms of the number of rules required, yet general and accurate in representing unique spatial relationships taking the complex effect of the size and proximity of the objects into account. The method can be implementable for the derivation of spatial relations in spatial query languages.

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| Approach | Frame of reference | Space division | Defining relation | Object types | Validity on proximity-interaction axis |
|---|---|---|---|---|---|
| Peuquet and C-Xiang [PCX87] | extrinsic | conical | inside | point - extended object | disjoint - disjoint with overlapping MBR |
| Frank [Fra92] | extrinsic | rectangular | inside | point - extended object | disjoint |
| Hernandez [Her94] | Deictic extrinsic intrinsic | conical | inside | point - extended object | disjoint - overlap |
| Mukerjee and Joe [MJ90] | intrinsic | rectangular | inside | point object | disjoint |
| Freksa [Fre92] | Deictic | rectangular | inside | point object | disjoint |
| Papadias and Sellis [PS93] | extrinsic | rectangular | order $(<,=,>)$ | point - extended object | disjoint - overlap - contain |
| Jungert [Jun94] | extrinsic | conical | order $(<,=,>)$ | point (reference object) point or extended (primary object) | disjoint |
| Cui et al [CCR93] | extrinsic deictic | rectangular | order $(before)$ | point - extended object | disjoint |
| proposed | extrinsic deictic intrinsic | conical or rectangular | intersection | point - extended object | disjoint - overlap - contain |

Table 1: Comparison between different approaches for the representation of orientation relations.