Noise Estimation in Long-Range Matched-Filter Envelope Sonar Data

Robert Bareš, *Student Member, IEEE,* Dafydd Evans, and Stephen Long,

***Abstract*—In sonar signal processing when selecting a threshold for detection, it is necessary to consider the noise in the signal to achieve the desired rates of detection and false alarm. The clutter component of this noise, caused by scattering from environmental features, is often a limiting factor. This is particularly the case when active sonar systems operate in shallow water. Suitable modeling of clutter-limited data is therefore vital for accurate detection in such environments. This paper investigates the *K*, Weibull and log-normal distributions as models for clutter-limited matched filter envelope sonar data, obtained using FM chirp pulses in a shallow water environment. The models are evaluated using modified Kolmogorov-Smirnov and Anderson-Darling tests. Critical values for the upper-tail Anderson-Darling statistic applied to the *K*-distribution are estimated by Monte-Carlo simulation and tabulated here. Results show that the *K* and Weibull distributions provide a good model of noise in clutter-limited environments. However, the *K*-distribution provides a better fit in the tails, which is important for target detection. The Kolmogorov-Smirnov test is shown to be an unsuitable method of evaluating fit when the tail of a distribution is of greatest interest. We also show that the estimated shape parameter of the *K*-distribution does provide a simple means of identifying regions dominated by clutter.**

***Index Terms*—clutter, K-distribution, modeling, sonar signal analysis.**

I. INTRODUCTION

NOISE in matched filter envelope data is commonly assumed to have Rayleigh distribution, but this assumption is known to fail in clutter-limited environments, where scatterers are not homogeneously distributed.

Target signals are detected by estimating the noise distribution in the region of interest, then applying a threshold to the tail of this empirical distribution. Thresholds must be chosen carefully to maximize the probability of detection (P_d) while simultaneously keeping the probability of false alarm (P_{fa}) to a minimum [1]. Improved statistical models of the noise, particularly in the tails of the distribution, will reduce the number of false alarms and therefore provide more accurate performance prediction in difficult environments.

In 2002, Abraham and Lyons [2] proposed the *K*-distribution as a model for echo returns from a finite number of point scatterers. Experimental studies have demonstrated that the *K*-distribution does indeed improve on the standard Rayleigh model [2], [3].

The aim of this paper is to compare the performance of the Rayleigh, log-normal, Weibull, and K distributions when estimating sonar noise in long-range (clutter-limited) data. It should be noted that, as this paper focuses on noise estimation for decreased P_{fa}, the results are independent of target statistics.

To evaluate the performance of each model we analyzed matched filter envelope data, obtained by a long-range sonar system using FM chirp pulses operating in a shallow water environment. The returns from a total of 40 pings were processed in three separate frequency bands.

This paper begins with an outline of models for sonar noise estimation and methods of quantifying goodness-of-fit in Section II. Experimental results are then presented in Section III, with further discussion found in Section IV.

II. NOISE ESTIMATION

If the real and imaginary components of matched filter output have Gaussian distribution, the matched filter envelope has Rayleigh distribution. The *K*-distribution was originally developed for radar systems [4], to account for modulation in amplitude not adequately represented by the Rayleigh distribution. The *K*-distribution is characterized by its shape and scale parameters, denoted by ν and λ respectively, and reduces to the Rayleigh distribution in the limit as $\nu \to \infty$. Using the second moment of the distribution as the scale parameter [5], the cumulative distribution function (CDF) of the *K*-distribution can be written as

$$F(x;\nu,\lambda) = 1 - \frac{2}{\Gamma(\nu)}\left(\frac{\nu}{\lambda}\right)^{\frac{\nu}{2}} x^{\nu} K_{\nu}\left(2x\left(\frac{\nu}{\lambda}\right)^{\frac{1}{2}}\right) \quad (1)$$

where $K_\nu(\cdot)$ is the modified Bessel function of the second kind. Beyond physically justified models of sonar data, distributions are often chosen for the simple reason that they fit the data. Sonar clutter is commonly seen to exhibit a heavy tailed matched filter envelope, as noted in [2]. Therefore the log-normal and Weibull distributions are often considered, both of which have heavier tails than the Rayleigh distribution. In particular, the Weibull distribution is part of the extreme value family of distributions, and includes both the Rayleigh and exponential distributions as special cases.

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R. Bareš and D. Evans are with the School of Computer Science, Cardiff University, Cardiff CF24 3AA, UK (e-mail: r.bares@ieee.org; d.evans@cs.cardiff.ac.uk).

S. Long is with Thales Underwater Systems, Cheadle Heath, Stockport SK3 0XB, U.K. (e-mail: stephen.long@uk.thalesgroup.com).

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The Rayleigh, log-normal, two-parameter Weibull, and *K* distributions will be considered in the following analysis. Maximum Likelihood Estimators (MLE) were used to estimate all distribution parameters, except for the shape parameter ν of the *K*-distribution.

As noted previously in the published literature [2], [6] the MLE of the *K*-distribution shape parameter is computationally intensive, requiring a numerical search and extensive calculation of the Bessel function $K_\nu(\cdot)$ [7]. This makes the MLE inappropriate for analysis of large data sets such as ours.

Therefore, in this work we use the iterative Method of Moments Estimator (MME) proposed in [2], with an initial seed provided by the $z\log(z)$ estimator of [5]. The $z\log(z)$ seed provides an overall lower variance estimate than the gamma approximation used in [2], particularly for low values of ν. This method of parameter estimation is far more tractable than the MLE for the *K*-distribution and is comparable in runtime to the MLE for the Weibull distribution.

Unfortunately, the iterative MME does not always yield a meaningful result. This is most often seen when the value of ν is large, leading to high variance as shown by the Cramér-Rao lower bound [2]. To compound this difficulty, numerical algorithms for computing the Bessel function $K_\nu(\cdot)$, which are needed to compute the *K*-distribution, become unstable at large values of ν. We avoid these difficulties by simply using a Rayleigh distribution whenever $\nu > 30$, employing the following relation for the Rayleigh scale parameter σ,

$$\sigma = \sqrt{\lambda/2}. \tag{2}$$

A. Goodness-of-Fit

To quantify the extent to which our data can be represented by a candidate distribution, we use modifications of the well-known Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) goodness-of-fit tests. Let $x_1, \ldots, x_n$ represent the ordered data samples. The KS test statistic [8] is defined to be the maximum absolute difference between the empirical distribution function $F_n(x)$ of the observed data and the cumulative distribution function $F(x)$ being tested,

$$D_n = \sup_x |F_n(x) - F(x)| \text{ where } F_n(x) = \frac{1}{n}\sum_{i=1}^{n} I(X_i \le x) \tag{3}$$

and $I(X_i \le x)$ is the indicator function. For a given significance level, if the computed value of D_n exceeds the critical value under the null hypothesis that the observed data is from distribution F, then the null hypothesis is rejected and we conclude that the data does not come from distribution F. While the KS test assumes that $F(x)$ is completely specified, we rely on *estimates* of the distribution parameters and must therefore adopt Lilliefors' modification [9] of the KS test when computing critical values.

Since the majority of sample points are observed near the center of a distribution, the KS statistic is relatively insensitive to deviations in the tails. In the present context, this is a serious drawback because the main reason for seeking improved noise estimates is to improve target detection, which invariably occurs in the upper tail of the observed distribution. To address this problem, we also use the Anderson-Darling goodness-of-fit test [10], [11], which gives more weight to observations in the tails of the distribution. As we are interested in only the upper tail of the distribution, we apply the *upper-tail* Anderson-Darling test [12], defined by

$$A_n^2 = \frac{n}{2} - 2\sum_{i=1}^{n} F(x_i) - \sum_{i=1}^{n}\left(2 - \frac{2i-1}{n}\right)\log(1 - F(x_i)) \tag{4}$$

The distribution of Lilliefors' KS statistic and the upper-tail AD statistic depend on the distribution function under investigation, the number of sample points n, and on whether the distribution parameters are estimated from the data [9], [11]. Tables of critical values for the distribution of the (two-tailed) AD statistic for a number of different distributions have previously appeared in the literature [13]–[16]. To the best of our knowledge, such tables have not been published for the upper-tail AD test applied to the *K*-distribution. Equally there are no known tables of critical values for Lilliefors' KS test applied to the *K* and Weibull distributions. We have therefore performed Monte Carlo simulations to estimate these critical values over a range of different shape parameter values, sample sizes and significance levels. Since the distributions of A_n^2 and Lilliefors' D_n are known to be scale-invariant, we define the scale parameter to be $\lambda = 1$ throughout [17]. The procedure is outlined as follows:

1) For a given shape parameter ν generate a sample of n *K*-distributed random values.
2) Compute estimates of the distribution parameters for each set of sample points.
3) Compute the test statistic using the sample points and the parameter estimates, binning the value of the statistic against the nearest shape parameter under consideration.
4) Repeat steps 1) to 3) to obtain 100,000 realizations of the statistic for each shape parameter value.
5) Compute the empirical distribution function of these 100,000 realizations.
6) Estimate the critical values corresponding to the required significance levels.

The results for A_n^2 are shown in Tables IV and V, for samples of size $n = 1000$ and $n = 100$ respectively. A summary of critical values for Lilliefors' D_n applied to the *K* and Weibull distributions can be found in Tables VI and VII.

For the *K*-distribution, critical values are seen to decrease as ν increases, which is to be expected as the distribution becomes less sensitive to ν. It should be noted that the *K*-distribution values were generated using the random number generator from [7], and the critical values are only valid if ν is estimated by the iterative MME seeded with the $z\log z$ estimator of [5].

Although Table V is not used in our analysis of the data, comparing Table IV with Table V clearly shows that the distribution of A_n^2 depends on the sample size n.

III. ANALYSIS OF TRIAL DATA

Our data was obtained by a long-range active sonar system in a shallow water environment. This trial data was gathered in the southwest approaches of the United Kingdom, a region known to contain variable levels of clutter and a large number of wrecks, and where the typical water depth is between 120 and 150 meters. The system consisted of a towed transmitter and a separate towed array, providing a nested 3 octave line array. For the 40 pings analyzed here, the array was towed in a straight line and used FM chirp pulses with no resolution of left-right ambiguity. Amplitude returns for each of the pings were calculated by the sonar processing chain. The processing included time-domain beamforming, basebanding, filtering and matched-filtering in three frequency bands. The time-domain beamformer provided 64 beams from 0 to 180 degrees in azimuth, equally spaced in cosine. The three frequency bands covered 900Hz to 7.2kHz in frequency, with respective bandwidths of 900Hz, 1.6kHz and 3.8kHz [18], which we refer to as the Low, Medium and High frequency bands respectively.

For each ping of data, consecutive sample windows of $n = 1000$ adjacent data points were examined (in range), with a 75% overlap between consecutive windows. To examine predominantly reverberant and cluttered data, the data was processed from the direct blast to two-thirds of the sonar systems range. Distribution parameters were estimated in each window, and the associated cumulative distribution functions were then computed using the parameter estimates. The estimated distribution functions were compared against the observed distribution in each window, evaluated using Lilliefors' KS test and the upper-tail AD test. The results of these tests were averaged over all windows and across all pings in each frequency band.

A. Goodness-of-Fit

Reverberation from the seabed and clutter from seabed features often lead to a heavy tailed distribution of amplitude returns. This is related to the frequency of the ping, as higher frequencies are more quickly attenuated by the sea volume, leading to reduced returns and decreased range. Non-Gaussian properties that lead to heavy tailed non-Rayleigh data are therefore expected at low frequency, while predominantly Rayleigh data is anticipated at high frequency. In such scenarios, medium frequency offers a trade-off between range and noise.

For each distribution, Table I shows the percentage of sample windows for which the null hypothesis was not rejected by Lilliefors' KS test at the $\alpha = 0.05$ significance level. The table suggests that the *K*-distribution, Weibull and Log-normal distributions offer improved performance over the Rayleigh distribution across all frequency bands. The improvement is particularly evident in the low frequency band, where the data exhibits high levels of clutter, but there are also small regions of clutter seen at high frequency which are not fitted by the Rayleigh distribution.

Similarly, Table II shows the percentage of sample windows for which the null hypothesis was not rejected by the upper-tail AD test, where the critical values for the *K*-distribution are computed from Table IV by interpolation. The results show that the *K* and Weibull distributions provide a consistently better fit to the tail of the data than the Rayleigh distribution. However, across all 3 frequency bands the *K*-distribution provided a better fit than the Weibull distribution. Despite the heavy-tailed nature of the Weibull distribution, by applying the two-tailed Anderson-Darling test it was observed that the Weibull distribution provided the best fit to the lower tail of the data, which is of little interest in the present context. In addition, the log-normal distribution was also observed to provide a poor fit to the tail of the data.

For both statistical tests and for every candidate distribution, the estimated distributions were seen to provide progressively better fits to the data as range increased. In this trial data it was also observed that the best fits occurred in broadside beams, with a steady deterioration as the beams approached the fore and aft directions.

B. Non-Rayleigh Environments

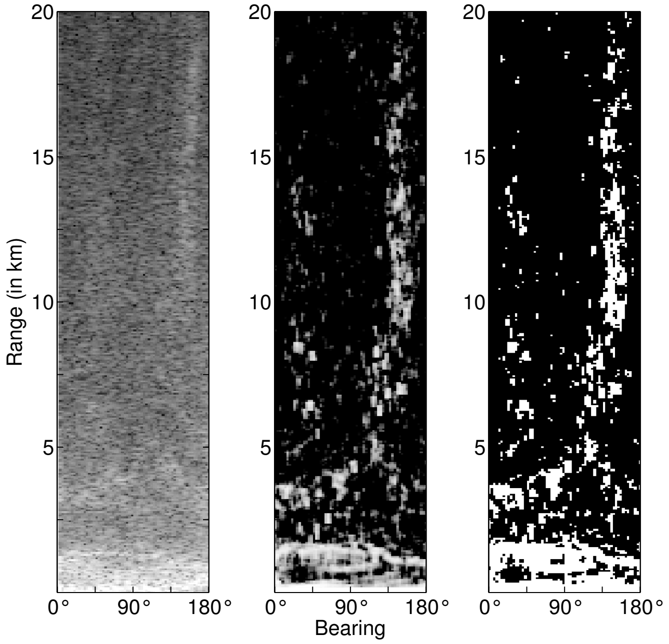
The relationship between the distribution parameters and environmental factors was investigated, where we found that scale parameters offered little information over observing the raw data. Additionally, the Weibull shape parameter behaved similarly to the log-normal variance and provided no significant information. However the *K*-distribution shape parameter (ν) was seen to be sensitive to regions of clutter; small values of ν were observed exclusively in cluttered, non-Rayleigh regions. This is to be expected, because the *K*-distribution increasingly diverges from the Rayleigh distribution as ν decreases. Estimators for this parameter offer a means of identifying regions where the noise will be under-estimated by the Rayleigh model. This is shown in the range-bearing plots in Fig. 1.

If we assume that the Rayleigh model applies in regions of high clutter, we will inevitably underestimate the probability of false alarms P_{fa}. To investigate the effect on false alarm rates when applying the Rayleigh model to non-Rayleigh data, we compute a threshold x_α for the estimated Rayleigh distribution of a sample window at a given false alarm rate α, then compute the true false alarm rate that this threshold would achieve if the sample window contains *K* or Weibull distributed noise.

Under the assumption that the data has this Rayleigh distribution, the threshold corresponding to the probability of false alarm α is given by

$$x_\alpha = \sigma\sqrt{-2\log(1-\alpha)} \tag{5}$$

If the data actually has a hypothetical distribution $f(x;\theta)$, the probability of false alarm achieved with the threshold x_α is given by $1-F(x_\alpha;\theta)$, where F is the CDF of the distribution. These probabilities were calculated for the *K* and Weibull CDFs in each sample window, using the previously estimated parameters. They were then averaged over all windows and pings in each frequency band, and compared against five values of false alarm rate α from which the thresholds x_α were calculated. The results are shown in Table III. Note that these values are only of theoretical interest since we make the assumption that the *K* and Weibull distributions are correctly fit to the data, which is not necessarily the case.



(a) (b) (c)

Fig. 1. Demonstration of the K-distribution shape parameter highlighting regions of non-Rayleigh clutter. (a) Medium frequency matched filter envelope sonar data, (b) Shape parameter with threshold to show $\nu \leq 10$ where white denotes small values, (c) White sample windows where Lilliefors' KS test has rejected the Rayleigh hypothesis at 0.05 significance

TABLE I
PROPORTION OF NON-REJECTION FOR LILLIEFORS' KS TEST AT 0.05 SIGNIFICANCE IN LOW, MEDIUM AND HIGH FREQUENCY BANDS (%).

Distribution	LF	MF	HF
K	94.09	99.22	99.38
Weibull	88.44	96.46	96.47
Log-normal	84.41	97.81	97.96
Rayleigh	72.01	88.75	89.71

TABLE II
PROPORTION OF NON-REJECTION FOR THE UPPER-TAIL AD TEST AT 0.05 SIGNIFICANCE IN LOW, MEDIUM AND HIGH FREQUENCY BANDS (%).

Distribution	LF	MF	HF
K	40.48	65.66	88.27
Weibull	39.20	60.62	84.72
Rayleigh	31.10	47.93	75.82
Log-normal	0.07	0.03	0.06

TABLE III
P_{fa} OBTAINED WHEN SELECTING A THRESHOLD BY ASSUMING RAYLEIGH DISTRIBUTED NOISE

	K-distribution			Weibull distribution		
Target P_{fa}	LF	MF	HF	LF	MF	HF
0.01	0.0149	0.0140	0.0138	0.0128	0.0129	0.0128
0.001	0.0033	0.0031	0.0030	0.0025	0.0031	0.0027
0.0001	0.0012	0.0013	0.0011	0.0010	0.0018	0.0013
0.00001	0.0006	0.0008	0.0006	0.0006	0.0014	0.0010
0.000001	0.0004	0.0005	0.0004	0.0005	0.0013	0.0009

IV. DISCUSSION

Compared to the standard Rayleigh model, we have shown that both the K and Weibull distributions offer improved models of clutter-limited matched filter envelope data.

As previously noted, Lilliefors' KS test is most sensitive to deviations in the center of the distribution. In contrast the upper-tail AD test is weighted to be sensitive to the right-hand tail, where detection occurs. The advantage of applying the upper-tail AD test can be seen by comparing the non-rejection rates of the Log-normal distribution in Tables I and II. The results of Lilliefors' KS test seen in Table I suggest that the Log-normal distribution provides a good fit to the center of the sample distribution. Yet in Table II the upper-tail AD test clearly demonstrates that the Log-normal distribution is a poor fit to the tail of the data. Considering this finding, Lilliefors' KS test should not be used alone to establish the suitability of a distribution when a good fit in the tail is important. This is particularly relevant to detection theory, as used in radar and sonar systems.

Using the upper-tail AD test, we have shown that the K and Weibull distributions are better able to model the tail of the observed data than the Rayleigh distribution, which is of great importance in performance prediction. Our analysis of the data has also shown that the K-distribution provides a better model of the tails than that provided by the Weibull distribution. While the Weibull distribution provides a reasonable fit to the noise data we must therefore conclude that, in addition to providing a useful physical model of sonar clutter, the K-distribution is also the most suitable model for accurate selection of detection thresholds. Additionally as a result of the compound nature of the K-distribution it may also be possible to extend the distribution to include further physical properties, in the same manner that the KA-distribution was obtained by modeling further properties of radar returns via the Class A model [19].

The negative effect of using the Rayleigh distribution to model data with non-Rayleigh properties was found to be most significant when setting low false alarm rates, which is often necessary in real-time applications where targets must be quickly identified. We have also shown that an estimate of the K-distribution shape parameter ν provides a simple way of identifying regions of high clutter in the data.

While upper-tail AD test critical values for the K-distribution were not derived analytically, the improved accuracy offered by such a derivation would be of considerable interest. Furthermore, while the techniques described in this paper involve noise estimation over a single sonar ping, the next stage of this research will be to investigate methods for clutter suppression over multiple pings, similar in concept to multi-look radar systems.

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TABLE IV
Upper-tail Anderson-Darling test critical values for the K-distribution, various significance levels, sample size 1000

	Significance level				
Shape	0.25	0.10	0.05	0.025	0.01
0.05	0.383	0.567	0.711	0.855	1.059
0.25	0.309	0.440	0.540	0.640	0.766
0.5	0.289	0.410	0.504	0.600	0.728
0.75	0.268	0.375	0.458	0.538	0.650
1	0.252	0.349	0.424	0.498	0.595
1.5	0.239	0.329	0.396	0.461	0.548
2	0.236	0.324	0.386	0.450	0.531
3	0.230	0.313	0.376	0.438	0.518
4	0.224	0.305	0.368	0.429	0.512
5	0.224	0.306	0.368	0.427	0.511
10	0.220	0.301	0.362	0.424	0.499
15	0.221	0.302	0.363	0.423	0.503
20	0.219	0.297	0.357	0.418	0.496
25	0.218	0.298	0.359	0.419	0.497
30	0.217	0.297	0.357	0.416	0.494

TABLE V
Upper-tail Anderson-Darling test critical values for the K-distribution, various significance levels, sample size 100

	Significance level				
Shape	0.25	0.10	0.05	0.025	0.01
0.05	0.368	0.541	0.683	0.822	1.016
0.25	0.278	0.386	0.465	0.543	0.644
0.5	0.259	0.357	0.431	0.504	0.603
0.75	0.241	0.329	0.395	0.462	0.548
1	0.235	0.318	0.380	0.444	0.528
1.5	0.227	0.308	0.368	0.431	0.506
2	0.222	0.303	0.363	0.424	0.501
3	0.221	0.301	0.362	0.421	0.504
4	0.222	0.301	0.362	0.422	0.499
5	0.221	0.300	0.360	0.421	0.498
10	0.219	0.300	0.359	0.421	0.498
15	0.216	0.293	0.353	0.413	0.489
20	0.213	0.290	0.347	0.404	0.480
25	0.213	0.291	0.349	0.406	0.486
30	0.215	0.294	0.354	0.413	0.491

TABLE VI
Lilliefors' KS test critical values for the K-distribution, various significance levels, sample size 1000

	Significance level				
Shape	0.25	0.10	0.05	0.025	0.01
0.05	0.740	0.754	0.762	0.770	0.779
0.25	0.256	0.275	0.286	0.297	0.310
0.5	0.082	0.092	0.098	0.105	0.113
1	0.055	0.060	0.062	0.065	0.068
5	0.043	0.047	0.049	0.052	0.054
10	0.041	0.045	0.047	0.050	0.052
20	0.040	0.044	0.046	0.048	0.051
30	0.040	0.043	0.046	0.048	0.050

TABLE VII
Lilliefors' KS test critical values for the Weibull distribution, various significance levels, sample size 1000

	Significance level				
Shape	0.25	0.10	0.05	0.025	0.01
0.1	0.993	0.996	0.997	0.998	0.998
0.25	0.924	0.944	0.956	0.964	0.973
0.5	0.559	0.602	0.629	0.655	0.685
0.75	0.230	0.255	0.272	0.289	0.310
1	0.082	0.092	0.098	0.105	0.113
1.25	0.051	0.055	0.058	0.061	0.064
1.5	0.043	0.047	0.050	0.052	0.055
3	0.038	0.042	0.044	0.047	0.049

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Robert Bareš (S08) received the B.Sc. (first class honors) degree in computer science with computer vision and computer graphics from Cardiff University, Cardiff, U.K., in 2007, where he is currently working towards the Ph.D. degree under an industrial CASE studentship funded by the EPSRC and Thales Underwater Systems.

During 2006, he was a Performance and Capacity Analyst at Siemens, Coventry, U.K. His current interests are in image processing, statistical signal processing, and remote sensing.

Dafydd Evans received the Ph.D. degree in mathematics from Cardiff University, Cardiff, U.K., in 2002.

Currently, he is a Royal Society University Research Fellow at the School of Computer Science, Cardiff University. His current research interests include statistical learning theory and modeling information propagation in opportunistic data networks.

Stephen Long received the Ph.D. degree in mathematics from the University of Wales, Aberystwyth, U.K., in 1992.

He has been working at Thales Underwater Systems, Cheadle Heath, U.K., since 1994. His current interests include sonar detection and tracking.