

# On extended conflict-freeness in argumentation

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## Abstract

This paper studies a possibility to represent  $n$ -ary conflicts within an argumentation framework having only binary attacks. We show that different instantiations of the abstract argumentation framework defined by Dung use very similar constructs for dealing with  $n$ -ary conflicts. We start by studying this procedure on two fully-instantiated systems from the argumentation literature and then show that it can also be formalised on the abstract level. We argue that this way of handling  $n$ -ary conflicts has two benefits. First, it allows to represent all the information within a standard argumentation framework, only by using arguments and attacks (e.g. without adding a new component to store the sets of conflicts). Second, all the added arguments have an intuitive interpretation, i.e. their meaning on the instantiated and on the abstract level is conceptually clear.

## 1 Introduction

The field of formal argumentation [6, 14] is based on the idea that reasoning can be performed by constructing and evaluating arguments, which are composed of a number of reasons that together support a claim. Arguments distinguish themselves from proofs by the fact that they are defeasible, that is, the validity of their conclusions can be disputed by other arguments. Whether a claim can be accepted therefore depends not only on the existence of an argument that supports this claim, but also on the existence of possible counter arguments, that can then themselves be attacked by counter arguments, etc.

This approach to reasoning has drawn a significant amount of attention since the conceptualisations of Pollock [12, 13], Vreeswijk [18], and Simari and Loui [16]. One of the common features of some of the formalisations in the 1990s (e.g. the work of Baroni et al. [4] or by Vreeswijk [18]) is the possibility to explicitly represent collective attacks between arguments. For example, in those frameworks, one is able to specify that there exist a set of three arguments  $\{A, B, C\}$  such that neither  $A$  nor  $B$  attack  $C$ , but  $A$  and  $B$  together attack  $C$ .

Nowadays, much research on the topic of argumentation is based on the theory proposed by Dung [10]. It allows one to abstract from the origin and the structure of arguments, by representing an argumentation framework as a directed graph, whose vertices correspond to arguments and arcs to attacks between them. However, this attack relation is binary, and it is not possible to explicitly specify  $n$ -ary attacks for  $n \geq 3$ . For instance, it is not possible to specify the existence of three arguments  $\{A, B, C\}$  such that neither  $A$  nor  $B$  attack  $C$ , but  $A$  and  $B$  together attack  $C$ .

Does this mean that it is impossible to represent ternary conflicts in such a setting? Are there instantiations of Dung's abstract theory that make it possible to specify that a set of arguments should not be accepted even if it is conflict-free with respect to a binary attack relation, i.e. even if there exist no arguments  $A, B$  in that set such that  $A$  attacks  $B$ ?

Take for example three different formulae  $\varphi, \psi, \omega$  such that the union of any two of those formulae is consistent and the union of all three formulae is inconsistent. Furthermore, let argument  $A$  be built by

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using only  $\varphi$ ,  $B$  by using only  $\psi$  and  $C$  by using only  $\omega$ . In virtually all instantiations of Dung's abstract argumentation theory, set  $\{A, B, C\}$  is conflict-free. However, there are ways to make sure that this set never appears as a part of an extension. The goal of this paper is to show that different instantiations of Dung's theory use very similar techniques to deal with this problem, which we refer to as "extended conflict-freeness". We will also argue that the technique used has two positive features. First, it allows to represent all the information about conflicts within an argumentation framework, without adding new components (such as a Boolean formula to represent a constraint [9] or a formula representing an acceptance condition for every argument [7]). Second, the added arguments have an intuitive interpretation, i.e. their meaning on the instantiated and on the abstract level is conceptually clear.

This paper is organised as follows: Section 2 defines the notions from argumentation theory needed for the present paper, Section 3 shows that different instantiations of Dung's theory from the argumentation literature use the same way to deal with extended conflict-freeness, Section 4 formalises this mechanism on the abstract level and Section 5 concludes.

## 2 Preliminaries

An argumentation framework is defined as a binary oriented graph, whose nodes represent arguments and whose arcs represent the attacks between them [10].

**Definition 1** An argumentation framework (*AF*) is a pair  $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ , where  $\mathcal{A}$  is a set of arguments and  $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$  is a binary relation representing the attacks between the arguments. For two arguments  $a, b \in \mathcal{A}$ , the notation  $aRb$  or  $(a, b) \in \mathcal{R}$  means that  $a$  attacks  $b$ .

The central notions in this theory are: conflict-freeness, defence and admissibility.

**Definition 2** Let  $\mathcal{F} = (\mathcal{A}, \mathcal{R})$  be an argumentation framework and  $\mathcal{S} \subseteq \mathcal{A}$  and  $a \in \mathcal{A}$ .

- $\mathcal{S}$  is conflict-free if and only if there is no  $a, b \in \mathcal{S}$  such that  $aRb$ .
- $\mathcal{S}$  defends argument  $a$  if and only if for every  $b \in \mathcal{A}$  if  $bRa$  then there exists  $c \in \mathcal{S}$  such that  $cRb$ .
- $\mathcal{S}$  is an admissible set in  $\mathcal{F}$  if and only if  $\mathcal{S}$  is conflict-free and defends all its elements.

A semantics is a function which, given an argumentation framework, calculates the sets of arguments which can be accepted together, called *extensions*. Let us now define some of the most commonly used semantics.

**Definition 3** Let  $\mathcal{F} = (\mathcal{A}, \mathcal{R})$  be an AF and  $\mathcal{S} \subseteq \mathcal{A}$ . We say that a set  $\mathcal{S}$  is admissible if and only if it is conflict-free and defends all its elements.

- $\mathcal{S}$  is a complete extension if and only if  $\mathcal{S}$  defends all its arguments and contains all the arguments it defends.
- $\mathcal{S}$  is a preferred extension if and only if it is a maximal (with respect to set inclusion) admissible set.
- $\mathcal{S}$  is a stable extension if and only if  $\mathcal{S}$  is conflict-free and for all  $a \in \mathcal{A} \setminus \mathcal{S}$ , there exists  $b \in \mathcal{S}$  such that  $bR a$ .
- $\mathcal{S}$  is a semi-stable extension if and only if  $\mathcal{S}$  is a complete extension and the union of the set  $\mathcal{S}$  and the set of all arguments attacked by  $\mathcal{S}$  is maximal (for set inclusion).
- $\mathcal{S}$  is a grounded extension if and only if  $\mathcal{S}$  is a minimal (for set inclusion) complete extension.
- $\mathcal{S}$  is an ideal extension if and only if  $\mathcal{S}$  is a maximal (for set inclusion) admissible set contained in every preferred extension.

**Definition 4** A semantics  $\sigma$  is admissibility-based if and only if for every argumentation framework  $\mathcal{F} = (\mathcal{A}, \mathcal{R})$  it holds that every extension of  $\mathcal{F}$  under semantics  $\sigma$  is an admissible set.

**Example 1** Complete, preferred, stable, semi-stable, grounded and ideal semantics are all admissibility-based.

### 3 Extended conflict-freeness in instantiated argumentation frameworks

In this section, we study the central question of the paper: how are  $n$ -ary conflicts handled in argumentation for  $n \geq 3$ ? Let us start with two examples. We start by examining two well-known approaches to instantiated argumentation: so-called “logic-based” approach, based on classical propositional logic [5], and so-called “rule-based” approach, which does not make use of propositional logic, but instead constructs arguments from rules of a given defeasible theory [8].

**Example 2** Suppose an instantiation of Dung’s theory with propositional logic where arguments are pairs (*support, conclusion*), support being a minimal consistent set of propositional formulae and conclusion being a formula such that  $\text{support} \vdash \text{conclusion}$ , where  $\vdash$  is the consequence operator from classical propositional logic [5]. Suppose a knowledge base  $\Sigma = \{x, y, \neg x \vee \neg y\}$ . Let  $A_1 = (\{x\}, x)$ ,  $A_2 = (\{y\}, y)$  and  $A_3 = (\{\neg x \vee \neg y\}, \neg x \vee \neg y)$  be three arguments and let  $S = \{A_1, A_2, A_3\}$ . Virtually all attack relations used in this setting satisfy conflict-dependence [1], that is, if an argument attacks another one, then the union of their supports is inconsistent. Furthermore, for any conflict-dependent relation, neither  $A_1$  attacks  $A_2$  nor  $A_2$  attacks  $A_1$ . The same holds for other pairs:  $A_1, A_3$  and  $A_2, A_3$ . Thus,  $S$  is conflict-free. However, under most of the existing semantics, one would like to ensure that no extension contains set  $S$ .

**Example 3** Suppose the ASPIC instantiation of Dung’s theory [8], where arguments are built from strict and defeasible rules. Suppose a defeasible theory  $\langle \mathcal{S}, \mathcal{D} \rangle$ , with  $\mathcal{S} = \{x, y \rightarrow \neg z; z, y \rightarrow \neg x; x, z \rightarrow \neg y\}$  and  $\mathcal{D} = \{\Rightarrow x; \Rightarrow y; \Rightarrow z\}$ . Let  $A_1 = (\Rightarrow x)$ ,  $A_2 = (\Rightarrow y)$  and  $A_3 = (\Rightarrow z)$  be three arguments and let  $S = \{A_1, A_2, A_3\}$ . Set  $S$  is conflict-free in this framework. However, one would like to avoid having an extension  $\mathcal{E}$  such that  $S \subseteq \mathcal{E}$ .

Both instantiations [5, 8] of Dung’s abstract theory mentioned in the previous examples ensure consistency by *constructing additional arguments*. How is this achieved? The answer is that one has to make sure that all the relevant arguments are constructed. The frameworks from Example 2 and 3 are not complete [17, Section 3] in the sense that not all arguments that can be constructed from the available knowledge are in the framework. Let us show how adding arguments solves the problem of modelling extended conflict-freeness with a binary attack relation.

**Example 4 (Example 2 Cont.)** The intuition in this example is that  $A_1$ ,  $A_2$  and  $A_3$  are not acceptable together. To be able to express this, one needs to create more arguments. We will construct an argument  $B_3$ , telling “ $A_1$  and  $A_2$  are in the extension, so  $A_3$  cannot be in the extension”. In other words, let  $B_3 = (\{x, y\}, \neg(\neg x \vee \neg y))$ . We can construct two more arguments telling that  $A_1$  and (respectively  $A_2$ ) cannot be in the extension since  $A_2$  and  $A_3$  (respectively  $A_1$  and  $A_3$ ) are in the extension:  $B_1 = (\{y, \neg x \vee \neg y\}, \neg x)$ ,  $B_2 = (\{x, \neg x \vee \neg y\}, \neg y)$ . Let us suppose that an argument  $X$  attacks argument  $Y$  if and only if there exists a formula  $\varphi$  in the support of  $Y$  such that the conclusion of  $X$  is logically equivalent to  $\neg\varphi$ . The corresponding argumentation graph is shown in Figure 1. This argumentation framework has four complete extensions  $\mathcal{E}_1 = \emptyset$ ,  $\mathcal{E}_2 = \{B_3, A_1, A_2\}$ ,  $\mathcal{E}_3 = \{B_1, A_2, A_3\}$  and  $\mathcal{E}_4 = \{B_2, A_1, A_3\}$ . There are three preferred extensions (that are also stable and semi-stable):  $\mathcal{E}_2$ ,  $\mathcal{E}_3$  and  $\mathcal{E}_4$ . The grounded extension coincides with the ideal extension and is equal to  $\mathcal{E}_1 = \emptyset$ . The main point is that “additional” arguments  $B_1, B_2, B_3$  allow the ternary conflict to be encoded in the argumentation graph.

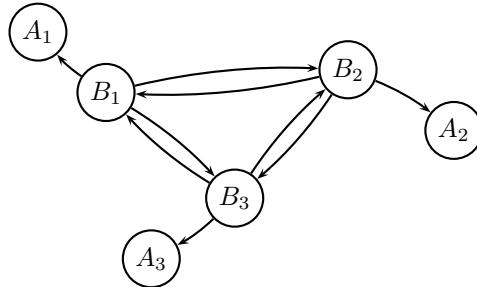


Figure 1: Arguments  $B_1$ ,  $B_2$  and  $B_3$  ensuring extended conflict-freeness

Note that other arguments can be constructed in Example 4, but they are not essential for the present discussion. Indeed, an infinite number of arguments can be constructed in this formalism, but it is known [2] that in this setting, for every infinite argumentation framework built from a finite number of propositional formulae, there exists an equivalent finite framework.

Let us see what happens in ASPIC [8] instantiation of Dung's theory.

**Example 5 (Example 3 Cont.)** Let  $B_3 = (A_1, A_2 \rightarrow \neg z)$ ,  $B_1 = (A_2, A_3 \rightarrow \neg x)$ , and  $B_2 = (A_1, A_3 \rightarrow \neg y)$ . The argument graph corresponding to this formalisation is the same as the graph from Example 4 (depicted in Figure 1).

Examples 4 and 5 show that the notion of extended conflict-freeness is already present in the literature, though not explicitly. It shows that existing instantiations construct argumentation framework in which the information about  $n$ -ary conflicts is encoded using a particular pattern in the graph.

## 4 Extended conflict-freeness on the abstract level

In this section, we abstract from the structure and contents of arguments and generalise the ideas presented in the previous section. Our goal is to show that every instantiation of Dung's abstract theory [10] can benefit from the same pattern which allows to deal with  $n$ -ary conflicts. We suppose that one is given an argumentation framework  $\mathcal{F} = (\mathcal{A}, \mathcal{R})$  and a collection  $S_1, \dots, S_n \subseteq \mathcal{A}$  of sets such that each set  $S_i$  represents a minimal conflict, in the sense that one does not want any extension to contain any of  $S_i$ -s. Note that this paper does not study the question *how to identify sets  $S_1, \dots, S_n$* . That question cannot be solved on the abstract level. We only identify and study a mechanism which ensures that extensions do not contain any of those sets, once the collection  $S_1, \dots, S_n$  is known. This is reflected in the next definition, which just accepts *any* collection of sets  $S_1, \dots, S_n \subseteq \mathcal{A}$ . Of course, the question how to identify  $S_1, \dots, S_n$  is a very important one, but is not a topic of this paper.

**Definition 5** Let  $\mathcal{F} = (\mathcal{A}, \mathcal{R})$  be an argumentation framework. A collection of minimal argumentation conflicts is a finite collection of sets  $\mathcal{C} = \{S_1, \dots, S_n\}$  such that for all  $i \in \{1, \dots, n\}$ , we have  $S_i \subseteq \mathcal{A}$ .

We now present on the abstract level a way used to ensure extended conflict freeness in many extended instantiations of Dung's theory. The idea is that for every minimal argumentation conflict  $S_i = \{A_1^i, \dots, A_{k_i}^i\}$ , one generates the additional arguments  $B_1, \dots, B_{k_i}$  such that

- each  $B_k^i$  attacks  $A_k^i$ ,
- each  $B_j^i$  also attacks every  $B_k^i$ , for  $j \neq k$ ,
- each attacker of  $A_k^i$  also attacks every  $B_j^i$ , for  $j \neq k$ .

**Example 6** To illustrate this idea, consider an argumentation framework  $\mathcal{F} = (\mathcal{A}, \mathcal{R})$  with  $\mathcal{A} = \{A_1, A_2, A_3, A_4, A_5, C\}$ ,  $\mathcal{R} = \{(C, A_1)\}$  and with minimal conflicts  $S_1 = \{A_1, A_2, A_3\}$  and  $S_2 = \{A_2, A_4, A_5\}$ . The extended framework  $\mathcal{F}' = (\mathcal{A}', \mathcal{R}')$ , with added arguments to model those conflicts, is depicted in Figure 2. Arguments  $B_1, B_2$  and  $B_3$  correspond to conflict  $S_1$ , whereas arguments  $B_2, B_4$  and  $B_5$  refer to  $S_2$ . Since  $C$  attacks  $A_1$  in the original framework, then it also attacks  $B_2$  and  $B_3$ .

The next definition formalises this procedure.

**Definition 6** Let  $\mathcal{F} = (\mathcal{A}, \mathcal{R})$  be an argumentation framework, let  $\sigma$  be an admissibility-based semantics and let  $\mathcal{C} = \{S_1, \dots, S_n\}$  be a collection of minimal argumentation conflicts. Let  $S_1 = \{A_1^1, \dots, A_{k_1}^1\}$ , ...  $S_i = \{A_1^i, \dots, A_{k_i}^i\}$  ...  $S_n = \{A_1^n, \dots, A_{k_n}^n\}$ .

The extended conflict-free version of  $\mathcal{F}$  with respect to  $\mathcal{C}$  is defined as  $\mathcal{F}' = (\mathcal{A}', \mathcal{R}')$ , where:

- $\mathcal{A}' = \mathcal{A} \cup \mathcal{B}_1 \cup \dots \cup \mathcal{B}_n$ , where  $\mathcal{B}_1 = \{B_1^1, \dots, B_{k_1}^1\}$ , ...  $\mathcal{B}_i = \{B_1^i, \dots, B_{k_i}^i\}$  ...  $\mathcal{B}_n = \{B_1^n, \dots, B_{k_n}^n\}$ .
- $\mathcal{R}' = \mathcal{R} \cup \mathcal{R}_1 \cup \mathcal{R}_2 \cup \mathcal{R}_3$ , where<sup>1</sup>  
 $\mathcal{R}_1 = \{(B_j^i, A_j^i) \mid i \in \{1, \dots, n\}, j \in \{1, \dots, k_i\}\}$ ,  
 $\mathcal{R}_2 = \{(B_j^i, B_l^i) \mid i \in \{1, \dots, n\}, j, l \in \{1, \dots, k_i\}, j \neq l\}$   
 $\mathcal{R}_3 = \{(C, B_j^i) \mid i \in \{1, \dots, n\}, j, l \in \{1, \dots, k_i\}, C \in \mathcal{A}, (C, A_l^i) \in \mathcal{R} \text{ and } j \neq l\}$ .

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<sup>1</sup>Note that the values of  $j$  and  $l$  depend on  $i$ , i.e.  $j = j(i)$  and  $l = l(i)$ .

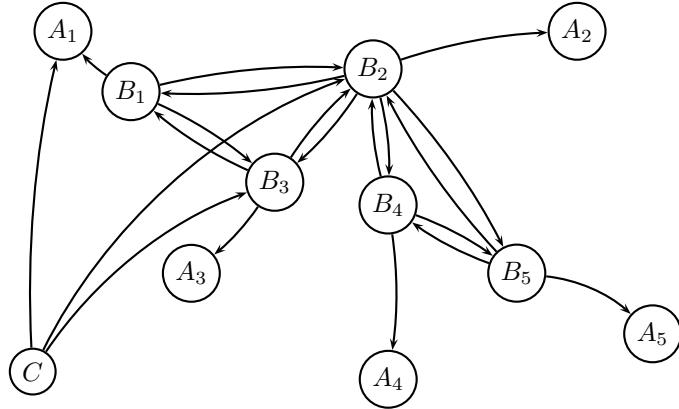


Figure 2: Extension of argumentation framework  $\mathcal{F}$  from Example 6 with auxiliary arguments and attacks

The previous definition shows how  $n$ -ary conflicts are translated into binary conflicts in various instantiations of Dung's theory.

Let us analyse this in detail. Arguments  $B_j^i$  are added to ensure consistency at the instantiated level. An argument  $B_j^i$  stands for: “there is a conflict  $S_i$ , and if you want to accept  $A_1^i, \dots, A_{j-1}^i, A_{j+1}^i, \dots, A_{k_i-1}^i$ , then you cannot accept  $A_j^i$ ”. To achieve this,  $B_j^i$  attacks  $A_j^i$ . Also, for every  $i$ , arguments  $B_1^i, \dots, B_{k_i}^i$  are mutually incompatible, since each of them relies on all but one arguments from a minimal argumentation conflict. Finally, what is the use of adding attacks from “ $C$  arguments”, i.e. from existing arguments to “ $B$  arguments”? The point is that we do not want to destroy existing incompatibilities between arguments. As an illustration, there may be an accepted attacker  $C$  of  $A_1$  in Example 3. In such a situation, one should just reject  $A_1$  and accept  $A_2$  and  $A_3$ . Let us construct such an example.

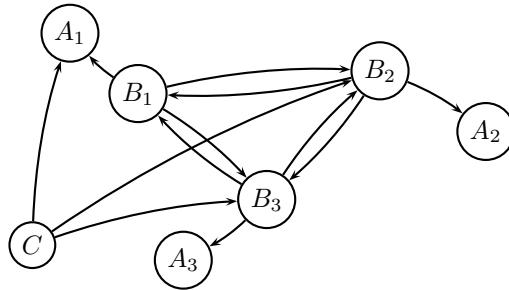


Figure 3: Argument  $C$  solving a ternary conflict: a correct solution

**Example 7 (Example 3 Contd.)** Let us suppose that the defeasible theory  $\langle \mathcal{S}, \mathcal{D} \rangle$ , is updated. Thus, we obtain a new theory  $\langle \mathcal{S}', \mathcal{D}' \rangle$ , with  $\mathcal{S}' = \mathcal{S} \cup \{\rightarrow t; t \rightarrow \neg[\Rightarrow x]\}$  and  $\mathcal{D}' = \mathcal{D}$ . Here,  $t$  is a fact such that  $x$  can be defeasibly concluded only in absence of  $t$ , i.e.  $t$  is an undercut of the defeasible rule allowing to conclude  $x$ . In addition to  $A_1, A_2, A_3, B_1, B_2, B_3$ , another significant argument can be constructed, namely  $C = ((\rightarrow t) \rightarrow \neg[\Rightarrow x])$ . The attack graph is depicted in Figure 3. There is a unique complete / stable / semi-stable / preferred / grounded / ideal extension:  $\{C, B_1, A_2, A_3\}$ . Intuitively, since  $A_1$  is undercut by  $C$  (which is undefeated) then  $A_1$  should not be accepted. So, there is no reason not to accept both  $A_2$  and  $A_3$ . In such a situation, forgetting to add attacks  $(C, B_2) \in \mathcal{R}'$  and  $(C, B_3) \in \mathcal{R}'$ , would result in a non-intuitive solution, as shown in Figure 4. The argumentation framework from Figure 4 has four complete extensions  $\mathcal{E}_1 = \{C\}$ ,  $\mathcal{E}_2 = \{C, B_1, A_2, A_3\}$ ,  $\mathcal{E}_3 = \{C, B_2, A_3\}$  and  $\mathcal{E}_4 = \{C, B_3, A_2\}$ . There are three stable / semi-stable / preferred extensions, namely  $\mathcal{E}_2$ ,  $\mathcal{E}_3$  and  $\mathcal{E}_4$ . The grounded extension coincides with the ideal extension and is equal to  $\mathcal{E}_1$ . Note that  $B_2$  stands for: “if you accept  $A_1$  and  $A_3$ , then you cannot

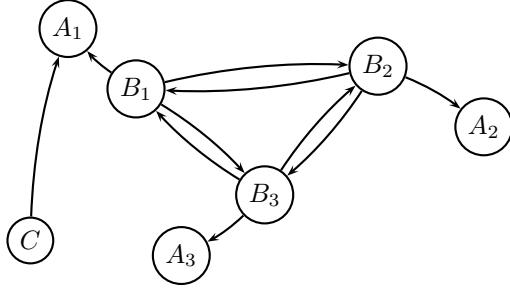


Figure 4: Argument  $C$  solving a ternary conflict: an incorrect solution

*accept  $A_2$ ". Informally, the conclusion of  $B_2$  is only valid in a situation when one does accept both  $A_1$  and  $A_3$ . But, since  $A_1$  is not accepted (because of  $C$ ), then arguments  $B_2$  and  $B_3$  "do not make sense": as soon as  $C$  is accepted, they should be rejected. Also, this produces two undesirable extensions:  $\mathcal{E}_3$  and  $\mathcal{E}_4$ . Namely, there is no reason to accept  $A_2$  and not  $A_3$  or vice versa, since there is no incompatibility whatsoever between  $A_2$  and  $A_3$ .*

The next theorem proves that the extensions of the extended conflict-free version of every argumentation framework satisfy extended conflict-freeness. Let us first formally define this notion.

**Definition 7** *Let  $\mathcal{F} = (\mathcal{A}, \mathcal{R})$  be an argumentation framework, let  $\sigma$  be an admissibility-based semantics and let  $\mathcal{C} = \{S_1, \dots, S_n\}$  be a collection of minimal argumentation conflicts. A set  $S \subseteq \mathcal{A}$  satisfies extended-conflict freeness if and only if there exists no  $S_i \in \mathcal{C}$  such that  $S_i \subseteq S$ .*

**Theorem 1** *Let  $\mathcal{F} = (\mathcal{A}, \mathcal{R})$  be an argumentation framework, let  $\sigma$  be an admissibility-based semantics and let  $\mathcal{C} = \{S_1, \dots, S_n\}$  be a collection of minimal argumentation conflicts. Let  $\mathcal{F}' = (\mathcal{A}', \mathcal{R}')$  be the extended conflict-free version of  $\mathcal{F}$ . Then: every extension of  $\mathcal{F}'$  under  $\sigma$  satisfies extended-conflict freeness.*

**Proof** *Let  $S_1 = \{A_1^1, \dots, A_{k_1}^1\}$ , ...  $S_i = \{A_1^i, \dots, A_{k_i}^i\}$  ...  $S_n = \{A_1^n, \dots, A_{k_n}^n\}$ . We will show that there is no extension containing a set  $S_i$ . To prove the theorem by reductio ad absurdum, suppose the contrary, i.e. let  $\mathcal{E}$  be an extension of  $\mathcal{F}'$  under semantics  $\sigma$ , let  $i \in \{1, \dots, n\}$  and let  $S_i \subseteq \mathcal{E}$ . Since  $\sigma$  is an admissibility-based semantics, then  $\mathcal{E}$  is an admissible set. This means that for every  $j \in \{1, \dots, k_i\}$ ,  $B_j^i \notin \mathcal{E}$ . Again from admissibility of  $\mathcal{E}$ , since for every  $j \in \{1, \dots, k_i\}$ , we have  $(B_j^i, A_j^i) \in \mathcal{R}'$ , then for every  $j \in \{1, \dots, k_i\}$  there exists  $C_j \in \mathcal{A}$  such that  $(C_j, B_j^i) \in \mathcal{R}'$  and  $C_j \in \mathcal{E}$  (informally speaking, arguments of "type B" can only be attacked by arguments of "type C"). Note that the case  $|S_i| = 1$  is not possible, since that would mean that  $B_1^i$  is not attacked in  $\mathcal{F}'$ . Thus,  $|S_i| \geq 2$ . This means that there exists  $A_2^i \in \mathcal{E}$ . From Definition 6, we have that  $(C_1, A_2^i) \in \mathcal{R}'$ . This would mean that set  $\mathcal{E}$  is not conflict-free. Contradiction with the facts  $A_2^i \in \mathcal{E}$ ,  $C_1 \in \mathcal{E}$  and that  $\mathcal{E}$  is an admissible set. This means that no extension contains a set  $S_i \in \mathcal{C}$ . ■*

## 5 Discussion, related literature and future work

In this paper, we showed how  $n$ -ary conflicts with  $n \geq 3$  are dealt with in different instantiations of Dung's abstract argumentation theory. To the best of our knowledge, this is the first paper pointing out that *different* instantiations of Dung's theory use very similar techniques to deal with this issue. We believe that this is one of the rare modelisations which at the same time allows to represent all the information about conflicts within an argumentation framework, without adding new components (e.g. a Boolean formula to represent a constraint or a formula representing an acceptance condition for every argument) and where the added arguments have an intuitive interpretation, i.e. their meaning on the instantiated and on abstract level is conceptually clear. In this section, we review the related work and show why we believe that some existing formalisations violate at least one of those two positive properties.

Brewka and Woltran [7] introduced *abstract dialectical frameworks* (ADFs), a generalisation of Dung's theory. They argue that the only interaction between arguments in Dung's framework is attack, and propose to attach to every argument an acceptance condition in form of a classical propositional logic formula,

using other arguments as atoms. The fact that  $A_1$  and  $A_2$  together attack  $A_3$  can be modelled by attaching the formula  $\neg(A_1 \wedge A_2)$  as the acceptance condition of argument  $A_3$ . They prove that for every ADF, there exists a Dung-style argumentation framework such that any *model* of the original ADF corresponds to a *stable extension* of the corresponding AF and vice versa. They also show that similar translations are possible between the *well-founded model* of an ADF and the *grounded extension* of an AF and between *stable models* of an ADF *stable extensions* of an AF. It is clear that ADFs are a more general tool (e.g. they also allow to represent supports) than the approach described in this paper. However, the approach we present can be used for every admissibility-based semantics without change, whereas the translation from an ADF to an AF was proved only for three semantics. Also, it is intuitively clear what is the meaning of every argument  $B_j^i$  that is added to the framework and which arguments should attack it, whereas when an ADF is translated to an AF, some arguments have purely technical meaning. A part of future work will also be to compare the robustness of the approach where an ADF is translated to an AF to represent  $n$ -ary attacks and the approach reported in the present paper. Namely, we want to explore how well the *dynamics of argumentation* is handled by the two approaches. This includes the capacity of an approach to be updated when a new  $n$ -ary attack is added, without having to recompute everything. We will also formally study the possibility of using constrained argumentation frameworks [9] for modelling  $n$ -ary attacks and compare those results with the results obtained by the approach described in the present paper. Once again, a benefit of using the present approach is that it is able to express all the conflicts on the basic AF level, whereas it generates only arguments with a clear conceptual meaning, i.e.  $A'_i$  stands for “*argument  $A_i$  cannot be accepted because all arguments  $A_1, \dots, A_{i-1}, A_{i+1}, \dots, A_n$  are already accepted*”.

Another related paper is the work [15] aimed at constructing *argumentation patterns*, such as conjunction, disjunction, or more complex constructs (e.g. Toulmin scheme). While the present paper’s goal is to understand how existing instantiated systems deal with  $n$ -ary attacks, the work of Villata et al. deals with situations where argumentation frameworks are not generated from a knowledge base, but where the knowledge engineer has to directly design arguments and attacks.

An important remark is that this paper shows the similarity between so called *logic-based instantiations* [3, 11] and *rule-based instantiations* [8] when it comes to generating particular patterns in the argumentation framework (as illustrated by Examples 4 and 5). Our future work will be to study *why* those patterns occur, are there other possibilities to model  $n$ -ary conflicts, and if yes, what is the “best” way to do it.

This paper presents the first step towards understanding how extended conflict-freeness is handled in instantiations of Dung’s framework. A part of our future work is to continue the formalisation and to prove its properties. For instance, we argue in Example 7 that  $C$  should attack  $B_2$  and  $B_3$  and explain the intuition behind such a definition. We plan to formalise these explications and prove what kind of properties the presented formalisation satisfies in general.

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