Computing Science

On the Difference between Assumption-Based Argumentation and Abstract Argumentation

Martin Caminada
Samy Sá
João Alcântara
Wolfgang Dvořák

Technical Report Series **ABDN–CS–1213XX**
Department of Computing Science **September 2013**
University of Aberdeen
King's College
Aberdeen AB24 3UE

On the Difference between Assumption-Based Argumentation and Abstract Argumentation

Martin Caminada
Samy Sá
João Alcântara
Wolfgang Dvořák

Technical Report ABDN–CS–1213XX
Department of Computing Science
University of Aberdeen

September 25, 2013

In the current paper, we re-examine the connection between abstract argumentation and assumption-based argumentation. Although these are often claimed to be equivalent, we observe that there exist well-studied admissibility-based semantics (semi-stable and eager) under which equivalence does not hold.

1 Introduction

The 1990s saw some of the foundational work in argumentation theory. This includes the work of Simari and Loui [17] that later evolved into Defeasible Logic Programming (DeLP) [13] as well as the ground-breaking work of Vreeswijk [20] whose way of constructing arguments has subsequently been applied in the various versions of the ASPIC formalism [6, 16, 15]. Two approaches, however, stand out for their ability to model a wide range of existing formalisms for non-monotonic inference. First of all, there is the abstract argumentation approach of Dung [11], which is shown to be able to model formalisms like Default Logic, logic programming under stable and well-founded model semantics [11], as well as Nute's Defeasible Logic [14] and logic programming under the 3-valued stable model semantics [21]. Secondly, there is the assumption-based argumentation approach of Bondarenko, Dung, Kowalski and Toni [2], which is shown to model formalisms like Default Logic, logic programming under stable model semantics, auto epistemic logic and circumscription [2].

One of the essential differences between these two approaches is that abstract argumentation is argument-based. One uses the information in the knowledge base to construct arguments and to examine how these arguments attack each other. Semantics is then defined on the resulting argumentation framework (the directed graph in which the nodes represent arguments and the arrows represent the attack relation). In assumption-based argumentation, on the other hand, semantics is defined based not on arguments but on sets of assumptions that attack each other based on their possible inferences.

One claim that occurs several times in the literature is that abstract argumentation and assumption-based argumentation are somehow equivalent. That is, the outcome (in terms of conclusions) of abstract argumentation would be the same as the outcome of assumption-based argumentation [10, 16]. In the current paper, we argue that although this equivalence does hold under *some* semantics, it definitely does not hold under *every* semantics. In particular, we show that under two well-known and well-studied admissibility-based semantics (semi-stable [19, 4, 7] and eager [5, 1, 12]) the outcome of assumption-based argumentation is fundamentally different from the outcome of abstract argumentation.

2 Preliminaries

Over the years, different versions of the assumption-based argumentation framework have become available [2, 9, 10] and these versions use slightly different ways of describing formal detail. For current purposes, we apply the formalization described in [10] which not only is the most recent, but is also relatively easy to explain.

Definition 1 ([10]). *Given a deductive system $\langle \mathcal{L}, \mathcal{R} \rangle$ where $\mathcal{L}$ is a logical language and $\mathcal{R}$ is a set of inference rules on this language, and a set of assumptions $\mathcal{A} \subseteq \mathcal{L}$, an* argument *for $c \in \mathcal{L}$ (the* conclusion *or* claim*) supported by $S \subseteq \mathcal{A}$ is a tree with nodes labelled by formulas in $\mathcal{L}$ or by the special symbol $\top$ such that:*

- *the root is labelled c*
- *for every node N*
  - *if N is a leaf then N is labelled either by an assumption or by $\top$*
  - *if N is not a leaf and b is the label of N, then there exists an inference rule $b \leftarrow b_1, \ldots, b_m$ ($m \geq 0$) and either $m = 0$ and the child of N is labelled by $\top$, or $m > 0$ and N has m children, labelled by $b_1, \ldots, b_m$ respectively*
- *S is the set of all assumptions labelling the leaves*

We say that a set of assumptions $\mathcal{A}sms \subseteq \mathcal{A}$ enables the construction of an argument A (or alternatively, that A can be constructed based on $\mathcal{A}sms$) if A is supported by a subset of $\mathcal{A}sms$.

Definition 2 ([10]). *An ABA framework is a tuple $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^- \rangle$ where:*

- *$\langle \mathcal{L}, \mathcal{R} \rangle$ is a deductive system*
- *$\mathcal{A} \subseteq \mathcal{L}$ is a (non-empty) set, whose elements are referred to as assumptions*
- *$^-$ is a total mapping from $\mathcal{A}$ into $\mathcal{L}$, where $\overline{\alpha}$ is called the contrary of α*

For current purposes, we restrict ourselves to ABA-frameworks that are *flat* [2], meaning that no assumption is the head of an inference rule. Furthermore, we follow [10] in that each assumption has a unique contrary.

We are now ready to define the various abstract argumentation semantics (in the context of an ABA-framework). We say that an argument A_1 *attacks* an argument A_2 iff the conclusion of A_1 is the contrary of an assumption in A_2. Also, if $\mathcal{A}rgs$ is a set of arguments, then we write $\mathcal{A}rgs^+$ for $\{A \mid$ there exists an argument in $\mathcal{A}rgs$ that attacks $A\}$. We say that a set of arguments $\mathcal{A}rgs$ is *conflict-free* iff $\mathcal{A}rgs \cap \mathcal{A}rgs^+ = \emptyset$. We say that a set of arguments $\mathcal{A}rgs$ *defends* an argument A iff each argument that attacks A is attacked by an argument in $\mathcal{A}rgs$.

Definition 3. *Let $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^- \rangle$ be an ABA framework, and let Ar be the associated set of arguments. We say that $\mathcal{A}rgs \subseteq Ar$ is:*

- *a* complete argument extension *iff $\mathcal{A}rgs$ is conflict-free and $\mathcal{A}rgs = \{A \in Ar \mid \mathcal{A}rgs$ defends $A\}$*

- *a* grounded argument extension *iff it is the minimal complete argument extension*
- *a* preferred argument extension *iff it is a maximal complete argument extension*
- *a* semi-stable argument extension *iff it is a complete argument extension where* $\mathcal{A}rgs \cup \mathcal{A}rgs^{+}$ *is maximal among all complete argument extensions*
- *a* stable argument extension *iff it is a complete argument extension where* $\mathcal{A}rgs \cup \mathcal{A}rgs^{+} = Ar$
- *an* ideal argument extension *iff it is the maximal complete argument extension that is contained in each preferred argument extension*
- *an* eager argument extension *iff it is the maximal complete argument extension that is contained in each semi-stable argument extension*

It should be noticed that the grounded argument extension is unique, just like the ideal argument extension and the eager argument extension are unique [5]. Also, every stable argument extension is a semi-stable argument extension, and every semi-stable argument extension is a preferred argument extension [4]. Furthermore, if there exists at least one stable argument extension, then every semi-stable argument extension is a stable argument extension [4]. It also holds that the grounded argument extension is a subset of the ideal argument extension, which in its turn is a subset of the eager argument extension [5].

The next step is to describe the various ABA semantics. These are defined not in terms of sets of arguments (as is the case for abstract argumentation) but in terms of sets of assumptions. A set of assumptions $\mathcal{A}sms_1$ is said to *attack* an assumption α iff $\mathcal{A}sms_1$ enables the construction of an argument for conclusion $\overline{\alpha}$. A set of assumptions $\mathcal{A}sms_1$ is said to attack a set of assumptions $\mathcal{A}sms_2$ iff $\mathcal{A}sms_1$ attacks some assumption $\alpha \in \mathcal{A}sms_2$. Also, if $\mathcal{A}sms$ is a set of assumptions, then we write $\mathcal{A}sms^{+}$ for $\{\alpha \in \mathcal{A} \mid \mathcal{A}sms \text{ attacks } \alpha\}$. We say that a set of assumptions $\mathcal{A}sms$ is *conflict-free* iff $\mathcal{A}sms \cap \mathcal{A}sms^{+} = \emptyset$. We say that a set of assumptions *defends* an assumption α iff each set of assumptions that attacks α is attacked by $\mathcal{A}sms$.

Apart from the ABA-semantics defined in [9], we also define semi-stable and eager semantics in the context of ABA.[1]

Definition 4. *Let $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ be an ABA framework, and let $\mathcal{A}sms \subseteq \mathcal{A}$. We say that $\mathcal{A}sms$ is:*

- *a* complete assumption extension *iff $\mathcal{A}sms \cap \mathcal{A}sms^{+} = \emptyset$ and $\mathcal{A}sms = \{\alpha \mid \mathcal{A}sms \text{ defends } \alpha\}$*
- *a* grounded assumption extension *iff it is the minimal complete assumption extension*
- *a* preferred assumption extension *iff it is a maximal complete assumption extension*
- *a* semi-stable assumption extension *iff it is a complete assumption extension where* $\mathcal{A}sms \cup \mathcal{A}sms^{+}$ *is maximal among all complete assumption extensions*
- *a* stable assumption extension *iff it is a complete assumption extension where* $\mathcal{A}sms \cup \mathcal{A}sms^{+} = \mathcal{A}$
- *an* ideal assumption extension *iff it is the maximal complete assumption extension that is contained in each preferred assumption extension*

[1] Please notice that our definitions are slightly different from the ones in [9] (as we define all semantics in terms of complete extensions) but equivalence is proved in the appendix.

- *an* eager assumption extension *iff it is the maximal complete assumption extension that is contained in each semi-stable assumption extension*

It should be noticed that the grounded assumption extension is unique, just like the ideal assumption extension and the eager assumption extension are unique. Also, every stable assumption extension is a semi-stable assumption extension, and every semi-stable assumption extension is a preferred assumption extension. Furthermore, if there exists at least one stable assumption extension, then every semi-stable assumption extension is a stable assumption extension. It also holds that the grounded assumption extension is a subset of the ideal assumption extension, which in its turn is a subset of the eager assumption extension. Formal proofs are provided in the appendix. For now, we observe that in the context of ABA, semi-stable and eager semantics are well-defined and have properties that are similar to their abstract argumentation variants (as described in [4, 5]).

3 Equivalence and Inequivalence

As can be observed from Definition 4 and Definition 3, the way assumption-based argumentation works is very similar to the way abstract argumentation works. In fact, there is a clear correspondence between these approaches, that allows one to convert ABA-extensions to abstract argumentation extensions, and vice versa.

Definition 5. *Let $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^{-} \rangle$ be an ABA framework, and let Ar be the set of all arguments that can be constructed using this ABA framework.*

- *We define* `Asms2Args` $: 2^{\mathcal{A}} \rightarrow 2^{Ar}$ *to be a function such that* `Asms2Args`$(\mathcal{A}sms) = \{A \in Ar \mid$ *A can be constructed based on $\mathcal{A}sms\}$*

- *We define* `Args2Asms` $: 2^{Ar} \rightarrow 2^{\mathcal{A}}$ *to be a function such that* `Args2Asms`$(\mathcal{A}rgs) = \{\alpha \in \mathcal{A} \mid \alpha$ *is an assumption occurring in an $A \in \mathcal{A}rgs\}$*

Theorem 6 ([9]). *Let $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^{-} \rangle$ be an ABA framework, and let Ar be the set of all arguments that can be constructed using this ABA framework.*

1. *If $\mathcal{A}sms \subseteq \mathcal{A}$ is a complete assumption extension, then* `Asms2Args`$(\mathcal{A}sms)$ *is a complete argument extension, and if $\mathcal{A}rgs \subseteq Ar$ is a complete argument extension, then* `Args2Asms`$(\mathcal{A}rgs)$ *is a complete assumption extension.*

2. *If $\mathcal{A}sms \subseteq \mathcal{A}$ is the grounded assumption extension, then* `Asms2Args`$(\mathcal{A}sms)$ *is the grounded argument extension, and if $\mathcal{A}rgs \subseteq Ar$ is the grounded argument extension, then* `Args2Asms`$(\mathcal{A}rgs)$ *is the grounded assumption extension.*

3. *If $\mathcal{A}sms \subseteq \mathcal{A}$ is a preferred assumption extension, then* `Asms2Args`$(\mathcal{A}sms)$ *is a preferred argument extension, and if $\mathcal{A}rgs \subseteq Ar$ is a preferred argument extension, then* `Args2Asms`$(\mathcal{A}rgs)$ *is a preferred assumption extension.*

4. *If $\mathcal{A}sms \subseteq \mathcal{A}$ is the ideal assumption extension, then* `Asms2Args`$(\mathcal{A}sms)$ *is the ideal argument extension, and if $\mathcal{A}rgs \subseteq Ar$ is the ideal argument extension, then* `Args2Asms`$(\mathcal{A}rgs)$ *is the ideal assumption extension.*

5. *If $\mathcal{A}sms \subseteq \mathcal{A}$ is a stable assumption extension, then* `Asms2Args`$(\mathcal{A}sms)$ *is a stable argument extension, and if $\mathcal{A}rgs \subseteq Ar$ is a stable argument extension, then* `Args2Asms`$(\mathcal{A}rgs)$ *is a stable assumption extension.*

Proof. Points 2 and 4 have been proved in [9], and point 5 has been proved in [18, Theorem 1],[2] so we only need to prove points 1 and 3.

1, first conjunct: Let $\mathcal{A}sms \subseteq \mathcal{A}$ be a complete assumption extension and let $\mathcal{A}rgs = \texttt{Asms2Args}(\mathcal{A}sms)$.

The fact that $\mathcal{A}sms$ is conflict-free (that is $\mathcal{A}sms \cap \mathcal{A}sms^+ = \emptyset$) means one cannot construct an argument based on $\mathcal{A}sms$ that attacks any assumption in $\mathcal{A}sms$.[3] Therefore, one cannot construct an argument based on $\mathcal{A}sms$ that attacks any argument based on $\mathcal{A}sms$. Hence, $\mathcal{A}rgs$ is conflict-free (that is, $\mathcal{A}rgs \cap \mathcal{A}rgs^+ = \emptyset$).

The fact that $\mathcal{A}sms$ defends itself means that $\mathcal{A}sms$ defends each assumption in $\mathcal{A}sms$. Hence, $\mathcal{A}sms$ defends each argument based on $\mathcal{A}sms$ (each argument in $\mathcal{A}rgs$). That is, $\mathcal{A}rgs$ defends itself.

The fact that each assumption defended by $\mathcal{A}sms$ is in $\mathcal{A}sms$ means that each argument whose assumptions are defended by $\mathcal{A}sms$ is in $\mathcal{A}rgs$. Hence, each argument defended by $\mathcal{A}rgs$ is in $\mathcal{A}rgs$.

Altogether, we have observed that $\mathcal{A}rgs$ is conflict-free and contains precisely the arguments it defends. That is, $\mathcal{A}rgs$ is a complete argument extension.

1, second conjunct: Let $\mathcal{A}rgs \subseteq Ar$ be a complete argument extension and let $\mathcal{A}sms = \texttt{Args2Asms}(\mathcal{A}rgs)$.

Suppose $\mathcal{A}sms$ is not conflict-free. Then it is possible to construct an argument based on $\mathcal{A}sms$ (say A) whose conclusion is the contrary of an assumption in $\mathcal{A}sms$. A cannot be an element of $\mathcal{A}rgs$ (otherwise $\mathcal{A}rgs$ would not be conflict-free). From the thus obtained fact that $A \notin \mathcal{A}rgs$, together with the fact that $\mathcal{A}rgs$ is a complete argument extension, it follows that $\mathcal{A}rgs$ does not defend A. But this is impossible, because $\mathcal{A}rgs$ does defend all assumptions in A. Contradiction. Therefore, $\mathcal{A}sms$ is conflict-free.

The fact that $\mathcal{A}rgs$ defends itself means that every $A \in \mathcal{A}rgs$ is defended by $\mathcal{A}rgs$, which implies that every assumption occurring in $\mathcal{A}rgs$ is defended by $\mathcal{A}rgs$, so every $\alpha \in \mathcal{A}sms$ is defended by $\mathcal{A}sms$. Hence, $\mathcal{A}sms$ defends itself.

The final thing to be shown is that $\mathcal{A}sms$ contains every assumption it defends. Suppose $\mathcal{A}sms$ defends $\alpha \in \mathcal{A}$. This means that for each argument B with conclusion $\overline{\alpha}$, $\mathcal{A}sms$ enables the construction of an argument C that attacks B. The fact that all assumptions in C are found in arguments from $\mathcal{A}rgs$ means that C is defended by $\mathcal{A}rgs$ (this is because $\mathcal{A}rgs$ defends all its arguments). The fact that $\mathcal{A}rgs$ is a complete argument extension then implies that $C \in \mathcal{A}rgs$. This means that $\mathcal{A}rgs$ defends the argument (say, A) consisting of the single assumption α. Hence, $A \in \mathcal{A}rgs$, so $\alpha \in \mathcal{A}sms$.

Altogether, we have observed that $\mathcal{A}sms$ is conflict-free and contains precisely the assumptions it defends. That is, $\mathcal{A}sms$ is a complete assumption extension.

3, first conjunct: Let $\mathcal{A}sms \subseteq \mathcal{A}$ be a preferred assumption extension and let $\mathcal{A}rgs = \texttt{Asms2Args}(\mathcal{A}sms)$.

From point 1, it then follows that $\mathcal{A}rgs$ is a complete assumption extension. Suppose, towards a contradiction, that $\mathcal{A}rgs$ is not a *maximal* complete argument extension. Then there exists a complete argument extension $\mathcal{A}rgs' \supsetneq \mathcal{A}rgs$. Let $\mathcal{A}sms' = \texttt{Args2Asms}(\mathcal{A}rgs')$. It then holds that $\mathcal{A}sms' \supsetneq \mathcal{A}sms$. Moreover, from point 1 it follows that $\mathcal{A}sms'$ is a complete assumption

[2]Please note that our definition of ideal and stable semantics is slightly different than in [9, 18] but equivalence is proven in the appendix.

[3]We abuse terminology a bit and say that argument A attacks assumption α iff the conclusion of A is $\overline{\alpha}$. Similarly, we say that a set of assumptions $\mathcal{A}sms$ defends an argument A iff it defends each assumption in A, and we say that a set of arguments $\mathcal{A}rgs$ defends an assumption α iff for each argument B with conclusion $\overline{\alpha}$, there is an argument $C \in \mathcal{A}rgs$ that attacks B.

extension. But this would mean that $\mathcal{A}sms$ is not a *maximal* complete assumption extension. Contradiction.

3, second conjunct: Let $\mathcal{A}rgs \subseteq Ar$ be a complete argument extension and let $\mathcal{A}sms = \texttt{Args2Asms}(\mathcal{A}rgs)$. From point 1, it then follows that $\mathcal{A}sms$ is a complete assumption extension. Suppose, towards a contradiction, that $\mathcal{A}sms$ is not a *maximal* complete assumption extension. Then there exists a complete assumption extension $\mathcal{A}sms' \supsetneq \mathcal{A}sms$. Let $\mathcal{A}rgs' = \texttt{Asms2Args}(\mathcal{A}sms')$. It then holds that $\mathcal{A}rgs' \supsetneq \mathcal{A}rgs$. Moreover, from point 1 it follows that $\mathcal{A}rgs'$ is a complete argument extension. But this would mean that $\mathcal{A}rgs$ is not a *maximal* complete argument extension. Contradiction.

□

Proposition 1. *When restricted to complete assumption extensions and complete argument extensions, the functions* `Asms2Args` *and* `Args2Asms` *become bijections and each other's inverses.*

Proof. Let $\mathcal{A}sms$ be a complete assumption extension and let $\mathcal{A}rgs$ be a complete argument extension. It suffices to prove statements (1) and (2) below.

1. $\texttt{Args2Asms}(\texttt{Asms2Args}(\mathcal{A}sms)) = \mathcal{A}sms$

   (a) Suppose $\alpha \in \mathcal{A}sms$. Then there exists an argument in $A \in \texttt{Asms2Args}(\mathcal{A}sms)$ consisting of a single assumption α. Therefore, $\alpha \in \texttt{Args2Asms}(\texttt{Asms2Args}(\mathcal{A}sms))$.

   (b) Suppose $\alpha \notin \mathcal{A}sms$ (assume without loss of generality that $\alpha \in \mathcal{A}$). Then there exists no argument in $\texttt{Asms2Args}(\mathcal{A}sms)$ that contains α. Therefore, $\alpha \notin \texttt{Args2Asms}(\texttt{Asms2Args}(\mathcal{A}sms))$.

2. $\texttt{Asms2Args}(\texttt{Args2Asms}(\mathcal{A}rgs)) = \mathcal{A}rgs$.

   (a) Suppose $A \in \mathcal{A}rgs$. Then all assumptions used in A will be in $\texttt{Args2Asms}(\mathcal{A}rgs)$. This means that A can be constructed based on $\texttt{Args2Asms}(\mathcal{A}rgs)$. Therefore, $A \in \texttt{Asms2Args}(\texttt{Args2Asms}(\mathcal{A}rgs))$.

   (b) Suppose $A \notin \mathcal{A}rgs$ (assume without loss of generality that $A \in Ar$). The fact that $\mathcal{A}rgs$ is a complete argument extension implies that A is not defended by $\mathcal{A}rgs$. Therefore, there exists an argument $B \in Ar$ that attacks A, such that $\mathcal{A}rgs$ contains no C that attacks B. Assume, without loss of generality, that B attacks A by having a conclusion $\overline{\beta}$, where β is an assumption used in A. Then $\mathcal{A}rgs$ cannot contain any argument that uses assumption β (otherwise, this argument would not be defended against B, so $\mathcal{A}rgs$ would not be a complete arguments extension). Therefore, $\beta \notin \texttt{Args2Asms}(\mathcal{A}rgs)$. This means that A cannot be constructed based on $\texttt{Args2Asms}(\mathcal{A}rgs)$. Therefore, $A \notin \texttt{Asms2Args}(\texttt{Args2Asms}(\mathcal{A}rgs))$

□

From Proposition 1, together with Theorem 6 and the fact that each preferred, grounded, stable, or ideal extension is also a complete extension, it follows that under complete, grounded, preferred, stable or ideal semantics, argument extensions and assumption extensions are one-to-one related.

The above results might cause one to believe that similar observations can also be made for other semantics. Unfortunately, this is not always the case.

Theorem 7. *Let $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ be an ABA framework, and let Ar be the set of all arguments that can be constructed using this ABA framework.*

1. *It is* not *the case that if $\mathcal{A}sms \subseteq \mathcal{A}$ is a semi-stable assumption extension, then* $\texttt{Asms2Args}(\mathcal{A}sms)$ *is a semi-stable argument extension, and it is not the case that if $\mathcal{A}rgs \subseteq Ar$ is a semi-stable argument extension, then* $\texttt{Args2Asms}(\mathcal{A}rgs)$ *is a semi-stable assumption extension.*

2. *It is* not *the case that if $\mathcal{A}sms \subseteq \mathcal{A}$ is an eager assumption extension, then* $\texttt{Asms2Args}(\mathcal{A}sms)$ *is an eager argument extension, and it is not the case that if $\mathcal{A}rgs \subseteq Ar$ is an eager argument extension, then* $\texttt{Args2Asms}(\mathcal{A}rgs)$ *is an eager assumption extension.*

Proof. Let $\mathcal{F}_{ex1} = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \overline{} \rangle$ be an ABA framework with $\mathcal{L} = \{a, b, c, e, \alpha, \beta, \gamma, \epsilon\}$, $\mathcal{A} = \{\alpha, \beta, \gamma, \epsilon\}$, $\overline{\alpha} = a$, $\overline{\beta} = b$, $\overline{\gamma} = c$, $\overline{\epsilon} = e$ and $\mathcal{R} = \{r_1, r_2, r_3, r_4, r_5\}$ as follows:

$$r_1: \quad c \leftarrow \gamma \qquad r_2: \quad a \leftarrow \beta \qquad r_3: \quad b \leftarrow \alpha \qquad r_4: \quad c \leftarrow \gamma, \alpha \qquad r_5: \quad e \leftarrow \epsilon, \beta$$

The following arguments can be constructed from this ABA framework.

- A_1, using the single rule r_1, with conclusion c and supported by $\{\gamma\}$
- A_2, using the single rule r_2, with conclusion a and supported by $\{\beta\}$
- A_3, using the single rule r_3, with conclusion b and supported by $\{\alpha\}$
- A_4, using the single rule r_4, with conclusion c and supported by $\{\gamma, \alpha\}$
- A_5, using the single rule r_5, with conclusion e and supported by $\{\epsilon, \beta\}$
- A_α, A_β, A_γ and A_ϵ, consisting of a single assumption α, β, γ and ϵ, respectively.

These arguments, as well as their attack relation, are shown in Figure 1.

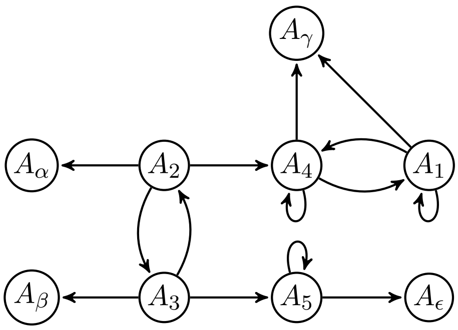

Figure 1: The argumentation framework AF_{ex1} associated with ABA framework $\mathcal{F}_{ex1}$.

The complete argument extensions of AF_{ex1} are $\mathcal{A}rgs_1 = \emptyset$, $\mathcal{A}rgs_2 = \{A_2, A_\beta\}$, and $\mathcal{A}rgs_3 = \{A_3, A_\alpha, A_\epsilon\}$. The associated complete assumption extensions of $\mathcal{F}_{ex1}$ are $\mathcal{A}sms_1 = \emptyset$, $\mathcal{A}sms_2 = \{\beta\}$, and $\mathcal{A}sms_3 = \{\alpha, \epsilon\}$. Notice that, as one would expect, $\mathcal{A}rgs_1 = \texttt{Asms2Args}(\mathcal{A}sms_1)$, $\mathcal{A}rgs_2 = \texttt{Asms2Args}(\mathcal{A}sms_2)$ and $\mathcal{A}rgs_3 = \texttt{Asms2Args}(\mathcal{A}sms_3)$, as well as $\mathcal{A}sms_1 = \texttt{Args2Asms}(\mathcal{A}rgs_1)$, $\mathcal{A}sms_2 = \texttt{Args2Asms}(\mathcal{A}rgs_2)$ and $\mathcal{A}sms_3 = \texttt{Args2Asms}(\mathcal{A}rgs_3)$.

It holds that $\mathcal{A}rgs_1 \cup \mathcal{A}rgs_1^+ = \emptyset$, $\mathcal{A}rgs_2 \cup \mathcal{A}rgs_2^+ = \{A_2, A_3, A_4, A_\alpha, A_\beta\}$ and $\mathcal{A}rgs_3 \cup \mathcal{A}rgs_3^+ = \{A_2, A_3, A_5, A_\alpha, A_\beta, A_\epsilon\}$, as well as $\mathcal{A}sms_1 \cup \mathcal{A}sms_1^+ = \emptyset$, $\mathcal{A}sms_2 \cup \mathcal{A}sms_2^+ = \{\alpha, \beta\}$ and $\mathcal{A}sms_3 \cup$

$\mathcal{A}sms_3^+ = \{\alpha, \beta, \epsilon\}$. Hence, $\mathcal{A}rgs_2$ and $\mathcal{A}rgs_3$ are semi-stable argument extensions, whereas only $\mathcal{A}sms_3$ is a semi-stable assumption extension. We thus have a counterexample against the claim that if $\mathcal{A}rgs$ ($\mathcal{A}rgs_2$) is a semi-stable argument extension, $\mathcal{A}sms = \texttt{Args2Asms}(\mathcal{A}rgs)$ ($\mathcal{A}sms_2$) is a semi-stable assumption extension.

We also observe that the eager argument extension is $\mathcal{A}rgs_1$ whereas the eager assumption extension is $\mathcal{A}sms_3$. Hence, we have a counterexample against the claim that if $\mathcal{A}rgs$ is an eager argument extension then $\mathcal{A}sms = \texttt{Args2Asms}(\mathcal{A}rgs)$ is an eager assumption extension, as well as against the claim that is $\mathcal{A}sms$ is an eager assumption extension then $\mathcal{A}rgs = \texttt{Asms2Args}(\mathcal{A}sms)$ is an eager argument extension.

The only thing left to be shown is that if $\mathcal{A}sms$ is a semi-stable assumption extension, then $\mathcal{A}rgs = \texttt{Asms2Args}(\mathcal{A}sms)$ is not necessarily a semi-stable argument extension. For this, we slightly alter the ABA framework $\mathcal{F}_{ex1}$ by removing rule r_5 and the assumption ϵ (call the resulting ABA framework $\mathcal{F}_{ex2}$). Thus the arguments A_5 and A_ϵ no longer exists and hence $\mathcal{A}rgs_3 = \{A_3, A_\alpha\}$. As now $\mathcal{A}rgs_3 \cup \mathcal{A}rgs_3^+ = \{A_2, A_3, A_\alpha, A_\beta\}$ is a proper subset of $\mathcal{A}rgs_2 \cup \mathcal{A}rgs_2^+$ the set $\mathcal{A}rgs_3$ is no longer semi-stable. On the other side both $\mathcal{A}sms_2 = \{\beta\}$, and $\mathcal{A}sms_3 = \{\alpha\}$ are semi-stable assumption extensions. □

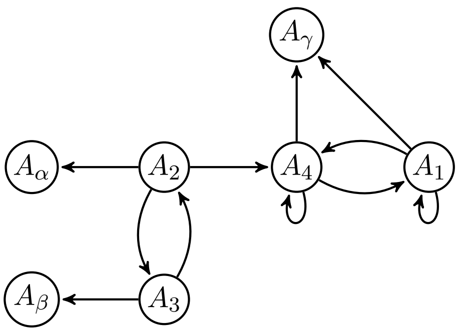

Figure 2: The argumentation framework AF_{ex2} associated with ABA framework $\mathcal{F}_{ex2}$.

4 Discussion

The connection between assumption-based argumentation and abstract argumentation has received quite some attention in the literature. Dung *et al.*, for instance, claim that "ABA is an instance of abstract argumentation (AA), and consequently it inherits its various notions of 'acceptable' sets of arguments" [10]. Similarly, Toni claims that "ABA can be seen as an instance of AA, and (...) AA is an instance of ABA" [18]. While we agree that this holds for *some* of the admissibility-based semantics (like preferred and grounded), we have pointed out in the current paper that this certainly does not hold for *all* admissibility-based semantics (semi-stable and eager). One could argue that claims like those above are perhaps a bit too general.

Prakken claims that "assumption-based argumentation (ABA) is a special case of the present framework [ASPIC+] with only strict inference rules, only assumption-type premises and no preferences." [16]. This claim is later repeated in the work of Modgil and Prakken, who state that "A well-known and established framework is that of assumption-based argumentation (ABA) [2], which (...) is shown (in [16])) to be a special case of the ASPIC+ framework in which arguments are built from assumption premises and strict inference rules only and in which all arguments are equally strong" [15]. However, we observe that the argumentation frameworks of Figure 1 and Figure 2 are counterexamples against this claim, in the context of semi-stable and eager semantics. These semantics, being admissibility-based, should work perfectly fine in the context of ASPIC+ (the rationality postulates of [6] would be satisfied). Nevertheless, correspondence with ABA does not hold.

A possible criticism against our counter example of Figure 1 is that it uses a rule (r_4) that is subsumed by another rule (r_1). This raises the quesion of whether counter examples still exist when no rule subsumes another rule. Our answer is affirmative: simply add an assumption δ and an atom d such that $\overline{\delta} = d$, replace r_1 by $c \leftarrow \gamma, \delta$ and add another rule (r_6) $d \leftarrow \delta$. For the resulting ABA theory, the semi-stable assumption extensions still do not correspond to the semi-stable argument extensions. Hence, the difference between ABA semi-stable (resp. ABA eager) and AA semi-stable (resp. AA eager) can be seen as a general phenomenon, that does not depend on whether some rules are subsumed by others.

Appendix: ABA semantics revisited

As mentioned earlier, the way the various ABA-semantics are defined in Definition 4 is slightly different from the way these were originally defined in [2, 9]. We have chosen to describe all ABA-semantics in a uniform way, based on the notion of complete semantics. This has been done not only for theoretical elegance, but also with an eye to possible future work. Ultimately, we would like to compare the various ABA-semantics to the various logic programming semantics, which in their turn can also be described in a uniform way using the concept of complete semantics (see [8, 3] for details).

We will now proceed to show that our description of ABA-semantics in Definition 4 is equivalent to the original description of ABA-semantics in [2, 9]. We start with preferred semantics. Notice that a set of assumptions is called *admissible* iff it is conflict-free and defends each of its elements.

Theorem 8. *Let $\mathcal{F} = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^{-} \rangle$ be an ABA framework. The following two statements are equivalent:*

1. *$\mathcal{A}sms$ is a maximal admissible assumption set of $\mathcal{F}$*
2. *$\mathcal{A}sms$ is preferred assumption extension of $\mathcal{F}$*

Proof. From 1 to 2: Let $\mathcal{A}sms$ be a maximal admissible assumption set. It follows from [2, Corollary 5.8] that $\mathcal{A}sms$ is a complete assumption extension. Suppose $\mathcal{A}sms$ is not *maximal* complete. Then there exists a complete assumption extension $\mathcal{A}sms'$ with $\mathcal{A}sms \subsetneq \mathcal{A}sms'$. But since by definition, every complete assumption extension is also an admissible assumption set, it holds that $\mathcal{A}sms'$ is an admissible assumption set. But this would mean that $\mathcal{A}sms$ is not a *maximal* admissible assumption set. Contradiction.
From 2 to 1: Let $\mathcal{A}sms$ be a maximal complete assumption extension. Then by definition, $\mathcal{A}sms$ is also an admissible assumption set. We now need to prove that it is also a *maximal* admissible assumption set. Suppose this is not the case, then there exists a maximal admissible assumption set $\mathcal{A}sms'$ with $\mathcal{A}sms \subsetneq \mathcal{A}sms'$. It follows from [2, Corollary 5.8] that $\mathcal{A}sms'$ is also a complete assumption extension. But this would mean that $\mathcal{A}sms$ is not a *maximal* complete assumption extension. Contradiction. □

The next thing to show is that our description of ideal semantics (Definition 4) coincides with that in [9]. More specifically, we will show that the notion of an ideal assumption extension is equivalent to that of a maximal ideal assumption set.

Definition 9. *Let $\mathcal{F} = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^{-} \rangle$ be an ABA framework. An* ideal assumption set *is defined as an admissible assumption set that is a subset of each preferred assumption extension.*

Lemma 1. *Let $\mathcal{F} = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^{-} \rangle$ be an ABA framework, and let $\mathcal{A}sms_{id}$ be a maximal ideal assumption set. It holds that $\mathcal{A}sms_{id}$ is a complete extension.*

Proof. Let $\mathcal{A}sms_{id}$ be a maximal ideal assumption set. We only need to prove that if $\mathcal{A}sms_{id}$ defends some $\alpha \in \mathcal{A}$ then $\alpha \in \mathcal{A}sms_{id}$. Suppose $\mathcal{A}sms_{id}$ defends α. Then every preferred assumption extension $\mathcal{A}sms_p$ also defends α (this follows from $\mathcal{A}sms_{id} \subseteq \mathcal{A}sms_p$). As $\mathcal{A}sms_p$ is also a complete extension, it follows that $\alpha \in \mathcal{A}sms_p$. Hence, α is an element of every preferred assumption extension. Therefore, $\mathcal{A}sms_{id} \cup \{\alpha\}$ is a subset of every preferred assumption extension. According to [2, Theorem 5.7], $\mathcal{A}sms_{id} \cup \{\alpha\}$ is also an admissible set. From the fact that $\mathcal{A}sms_{id}$ is a *maximal* ideal assumption set, and the trivial observation that $\mathcal{A}sms_{id} \subseteq \mathcal{A}sms_{id} \cup \{\alpha\}$, it then follows that $\mathcal{A}sms_{id} = \mathcal{A}sms_{id} \cup \{\alpha\}$. Therefore, $\alpha \in \mathcal{A}sms_{id}$. ☐

Theorem 10. *Let $\mathcal{F} = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^{-} \rangle$ be an ABA framework and let $\mathcal{A}sms \subseteq \mathcal{A}$. The following two statements are equivalent:*

1. *$\mathcal{A}sms$ is a maximal ideal assumption set of $\mathcal{F}$*
2. *$\mathcal{A}sms$ is an ideal assumption extension of $\mathcal{F}$ (in the sense of Definition 4)*

Proof. From 1 to 2: Let $\mathcal{A}sms$ be a maximal ideal assumption set. It follows from Lemma 1 that $\mathcal{A}sms$ is a complete assumption extension. Suppose $\mathcal{A}sms$ is not a *maximal* complete assumption extension that is contained in every preferred assumption extension. Then there exists a complete assumption extension $\mathcal{A}sms'$, with $\mathcal{A}sms \subsetneq \mathcal{A}sms'$, that is still contained in every preferred assumption extension. But since, by definition, every complete assumption extension is also an admissible assumption set, it holds that $\mathcal{A}sms'$ is an admissible assumption set that is contained in every preferred assumption extension. That is, $\mathcal{A}sms'$ is an ideal assumption set. But this would mean that $\mathcal{A}sms$ is not a *maximal* admissible assumption set. Contradiction.
From 2 to 1: Let $\mathcal{A}sms$ be an ideal assumption extension. Then, by definition, $\mathcal{A}sms$ is also an ideal assumption set. We now need to prove that it is also a *maximal* ideal assumption set. Suppose this is not the case, then there exists a maximal ideal assumption set $\mathcal{A}sms'$ with $\mathcal{A}sms \subsetneq \mathcal{A}sms'$. It follows from Lemma 1 that $\mathcal{A}sms'$ is also a complete assumption extension. But this would mean that $\mathcal{A}sms$ is not a *maximal* complete assumption extension that is contained in every preferred assumption extension. That is, $\mathcal{A}sms$ is not an ideal assumption extension. Contradiction. ☐

We proceed to show that our notion of stable semantics (Definition 4) coincides with the notion of stable semantics in [2].

Theorem 11. *Let $\mathcal{F} = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^{-} \rangle$ be an ABA framework, and let $\mathcal{A}sms \subseteq \mathcal{A}$. The following two statements are equivalent:*

1. *$\mathcal{A}sms$ does not attack itself and attacks each $\{\alpha\}$ with $\alpha \in \mathcal{A} \setminus \mathcal{A}sms$*
2. *$\mathcal{A}sms$ is a stable assumption extension of $\mathcal{F}$ (in the sense of Definition 4)*

Proof. From 1 to 2: Suppose $\mathcal{A}sms$ does not attack itself and attacks each $\{\alpha\}$ with $\alpha \in \mathcal{A} \setminus \mathcal{A}sms$. Then, according to [2, Theorem 5.5], $\mathcal{A}sms$ is a complete extension. Moreover, the fact that $\mathcal{A}sms$ attacks every $\{\alpha\}$ with $\alpha \in \mathcal{A} \setminus \mathcal{A}sms$ means that $\mathcal{A}sms \cup \mathcal{A}sms^{+} = \mathcal{A}$, so $\mathcal{A}sms$ is a complete extension with $\mathcal{A}sms \cup \mathcal{A}sms^{+} = \mathcal{A}$. That is, $\mathcal{A}sms$ is a stable extension.
From 2 to 1: Suppose $\mathcal{A}sms$ is a stable assumption extension. That is, $\mathcal{A}sms$ is a complete assumption

extension with $\mathcal{A}sms \cup \mathcal{A}sms^+ = \mathcal{A}$. From the fact that $\mathcal{A}sms$ is a complete assumption extension, it follows that $\mathcal{A}sms \cap \mathcal{A}sms^+ = \emptyset$ so $\mathcal{A}sms$ does not attack itself. From the fact $\mathcal{A}sms \cup \mathcal{A}sms^+ = \mathcal{A}$ it follows that $\mathcal{A}sms^+ = \mathcal{A} \setminus \mathcal{A}sms$, so $\mathcal{A}sms$ attacks each $\{\alpha\}$ with $\alpha \in \mathcal{A} \setminus \mathcal{A}sms$. □

So far, we have examined our characterization of existing ABA-semantics (stable, preferred and ideal semantics) and found them to be equivalent to what have been stated in the literature. The next step is to focus on the ABA-semantics that have not yet been stated in the literature[4] (semi-stable and eager). Our aim is to show that, in the context of ABA, these semantics behave in a very similar way as they do in the context of abstract argumentation. We start with the relation between stable, semi-stable and preferred semantics.

Theorem 12. *Let $\mathcal{F} = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^{-} \rangle$ be an ABA framework. It holds that:*

1. *every stable assumption extension is also a semi-stable assumption*
2. *every semi-stable assumption extension is also a preferred assumption extension*
3. *if there exists at least one stable assumption extension, then the stable assumption extensions and the semi-stable assumption extensions coincide*

Proof.

1. Let $\mathcal{A}sms$ be a stable assumption extension of $\mathcal{F}$. Then, by definition, $\mathcal{A}sms$ is a complete assumption extension with $\mathcal{A}sms \cup \mathcal{A}sms^+ = \mathcal{A}$. The fact that $\mathcal{A}sms \cup \mathcal{A}sms^+$ is $\mathcal{A}$ implies that it is maximal (by definition, it cannot be a proper superset of $\mathcal{A}$). Hence, $\mathcal{A}sms$ is a complete assumption extension where $\mathcal{A}sms \cup \mathcal{A}sms^+$ is maximal. That is, $\mathcal{A}sms$ is a semi-stable assumption extension.

2. Let $\mathcal{A}sms$ be a semi-stable assumption extension of $\mathcal{F}$. Them, by definition, $\mathcal{A}sms$ is a complete assumption extension where $\mathcal{A}sms \cup \mathcal{A}sms^+$ is maximal. We now show that $\mathcal{A}sms$ itself is also maximal. Suppose there is a complete assumption extension $\mathcal{A}sms'$ with $\mathcal{A}sms \subsetneq \mathcal{A}sms'$. Then, from the fact that the $^+$-operator is monotonic, it follows that $\mathcal{A}sms^+ \subseteq \mathcal{A}sms'^+$. This, together with the fact that $\mathcal{A}sms \subsetneq \mathcal{A}sms'$ implies that $\mathcal{A}sms \cup \mathcal{A}sms^+ \subsetneq \mathcal{A}sms' \cup \mathcal{A}sms'^+$. But that would mean that $\mathcal{A}sms$ is not a semi-stable assumption extension. Contradiction. Therefore, $\mathcal{A}sms$ is a maximal complete assumption extension. That is, $\mathcal{A}sms$ is a preferred assumption extension.

3. Suppose there exists at least one stable assumption extension ($\mathcal{A}sms_{st}$). The fact that every stable assumption extension is also a semi-stable assumption extension has already been proven by point 1, so the only thing left to prove is that every semi-stable assumption extension is also a stable assumption extension. Let $\mathcal{A}sms$ be a semi-stable assumption extension. Then, by definition, $\mathcal{A}sms$ is a complete assumption extension where $\mathcal{A}sms \cup \mathcal{A}sms^+$ is maximal. From the fact that $\mathcal{A}sms_{st}$ is a complete assumption extension with $\mathcal{A}sms_{st} \cup \mathcal{A}sms_{st}^+ = \mathcal{A}$, it follows that for $\mathcal{A}sms \cup \mathcal{A}sms^+$ to be maximal, it has to be $\mathcal{A}$ as well. This implies that $\mathcal{A}sms$ is a stable assumption extension.

□

We proceed to examine the concept of eager semantics in the context of ABA. Our aim is to show that the eager assumption extension is unique. In order to do so, we first need to define the concept of an eager assumption set. Notice that an eager assumption set relates to the eager assumption extension in the same way as an ideal assumption set relates to the ideal assumption extension.

[4] At least, not in the specific assumption-based ABA-context.

Definition 13. *Let $\mathcal{F} = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^{-} \rangle$ be an ABA framework. An* eager assumption set *is defined as an admissible assumption set that is a subset of each semi-stable assumption extension.*

Theorem 14. *Let $\mathcal{F} = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^{-} \rangle$ be an ABA framework. There exists precisely one maximal eager assumption set.*

Proof. We first prove that there exists at least one maximal eager assumption set. This is relatively straightforward, because there exists at least one eager assumption set (the empty set), which together with the fact that that there are only finitely many eager assumption sets (which follows from the fact that $\mathcal{A}$ is finite) implies that there exists at least one maximal eager assumption set.
The next thing to prove is that there exists at most one maximal eager assumption set. Let $\mathcal{A}sms_1$ and $\mathcal{A}sms_2$ be maximal eager assumption sets. From the fact that for each semi-stable assumption extension $\mathcal{A}sms_{sem}$, it holds that $\mathcal{A}sms_1 \subseteq \mathcal{A}sms_{sem}$ and $\mathcal{A}sms_2 \subseteq \mathcal{A}sms_{sem}$ it follows that $\mathcal{A}sms_1$ and $\mathcal{A}sms_2$ do not attack each other (otherwise $\mathcal{A}sms_{sem}$ would attack itself). Hence, $\mathcal{A}sms_3 = \mathcal{A}sms_1 \cup \mathcal{A}sms_2$ does not attack itself. Also, $\mathcal{A}sms_3$ defends itself, as $\mathcal{A}sms_1$ and $\mathcal{A}sms_2$ defend themselves. Hence, $\mathcal{A}sms_3$ is an admissible assumption set that is a subset of each semi-stable assumption extension. That is, $\mathcal{A}rgs_3$ is an eager assumption set. Also, from the fact that $\mathcal{A}sms_3 = \mathcal{A}sms_1 \cup \mathcal{A}sms_2$, it follows that $\mathcal{A}sms_1 \subseteq \mathcal{A}sms_3$ and $\mathcal{A}sms_2 \subseteq \mathcal{A}sms_3$. From the fact that $\mathcal{A}sms_1$ and $\mathcal{A}sms_2$ are *maximal* eager assumption sets, it then follows that $\mathcal{A}sms_1 = \mathcal{A}sms_3$ and $\mathcal{A}sms_2 = \mathcal{A}sms_3$. Therefore, $\mathcal{A}sms_1 = \mathcal{A}sms_2$. □

Lemma 2. *Let $\mathcal{F} = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^{-} \rangle$ be an ABA framework, and let $\mathcal{A}sms_{eag}$ be the maximal eager assumption set. It holds that $\mathcal{A}sms$ is a complete assumption extension.*

Proof. Let $\mathcal{A}sms_{eag}$ be a maximal eager assumption set. We only need to prove that if $\mathcal{A}sms_{eag}$ defends some $\alpha \in \mathcal{A}$ then $\alpha \in \mathcal{A}sms_{eag}$. Suppose $\mathcal{A}sms_{eag}$ defends α. Then every semi-stable assumption extension $\mathcal{A}sms_{sem}$ also defends α (this follows from $\mathcal{A}sms_{eag} \subseteq \mathcal{A}sms_{sem}$). As $\mathcal{A}sms_{sem}$ is also a complete assumption extension, it follows that $\alpha \in \mathcal{A}sms_{sem}$. Hence, α is an element of every semi-stable assumption extension. Therefore, $\mathcal{A}sms_{eag} \cup \{\alpha\}$ is a subset of every semi-stable assumption extension. According to [2, Theorem 5.7], $\mathcal{A}sms_{eag} \cup \{\alpha\}$ is also an admissible assumption set. Hence, $\mathcal{A}sms_{eag} \cup \{\alpha\}$ is an eager assumption set. From the fact that $\mathcal{A}sms_{eag}$ is a *maximal* eager assumption set, and the trivial observation that $\mathcal{A}sms_{eag} \subseteq \mathcal{A}sms_{eag} \cup \{\alpha\}$, it then follows that $\mathcal{A}sms_{eag} = \mathcal{A}sms_{eag} \cup \{\alpha\}$. Therefore, $\alpha \in \mathcal{A}sms_{eag}$. □

Theorem 15. *Let $\mathcal{F} = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, ^{-} \rangle$ be an ABA framework and let $\mathcal{A}sms \subseteq \mathcal{A}$. The following two statements are equivalent:*

1. *$\mathcal{A}sms$ is a maximal eager assumption set of $\mathcal{F}$*
2. *$\mathcal{A}sms$ is an eager assumption extension of $\mathcal{F}$ (in the sense of Definition 4)*

Proof. From 1 to 2: Let $\mathcal{A}sms$ be a maximal eager assumption set. It follows from Lemma 2 that $\mathcal{A}sms$ is a complete assumption extension. Suppose $\mathcal{A}sms$ is not a *maximal* complete assumption extension that is contained in every semi-stable assumption extension. Then there exists a complete assumption extension $\mathcal{A}sms'$, with $\mathcal{A}sms \subsetneq \mathcal{A}sms'$, that is still contained in every semi-stable assumption extension. But since by definition, every complete assumption extension is also an admissible assumption set, it holds that $\mathcal{A}sms'$ is an admissible assumption set that is contained in every semi-stable assumption extension. That is, $\mathcal{A}sms'$ is an eager assumption set. But this would mean that $\mathcal{A}sms$ is not a *maximal* eager assumption set. Contradiction.
From 2 to 1: Let $\mathcal{A}sms$ be an eager assumption extension. Then, by definition, $\mathcal{A}sms$ is also an eager

assumption set. We now need to prove that it is also a *maximal* eager assumption set. Suppose this is not the case, then there exists a maximal eager assumption set $\mathcal{A}sms'$ with $\mathcal{A}sms \subsetneq \mathcal{A}sms'$. It follows from Lemma 2 that $\mathcal{A}sms'$ is also a complete assumption extension. But this would mean that $\mathcal{A}sms$ is not a *maximal* complete assumption extension that is contained in every semi-stable assumption extension. That is, $\mathcal{A}sms$ is not an eager assumption extension. Contradiction. □

From the above observed fact that the eager assumption extension is unique (just like the ideal and grounded assumption extensions are unique), together with the fact that every semi-stable assumption extension is a preferred assumption extension, and every preferred assumption extension is a complete assumption extension, it follows that the grounded assumption extension is a subset of the ideal assumption extension, which is in its turn a subset of the eager assumption extension. Overall, we observe that in ABA context, semi-stable and eager semantics are well-defined and have properties that are similar to their abstract argumentation variants (as described in [4, 5]).

Acknowledgements

The first author has been supported by the National Research Fund, Luxembourg (LAAMI project) and by the Engineering and Physical Sciences Research Council (EPSRC, UK), grant ref. EP/J012084/1 (SAsSy project). The second and third authors have been supported by CNPq (Universal 2012 - Proc. n 473110/2012-1), CAPES (PROCAD 2009) and CNPq/CAPES (Casadinho/PROCAD 2011).

References

[1] P. Baroni, M.W.A. Caminada, and M. Giacomin. An introduction to argumentation semantics. *Knowledge Engineering Review*, 26(4):365–410, 2011.

[2] A. Bondarenko, P.M. Dung, R.A. Kowalski, and F. Toni. An abstract, argumentation-theoretic approach to default reasoning. *Artificial Intelligence*, 93:63–101, 1997.

[3] M.W.A. Caminada, S. Sá, and J. Alcântara. On the equivalence between logic programming semantics and argumentation semantics. In *Proceedings ECSQARU 2013*, 2013. in print.

[4] M.W.A. Caminada. Semi-stable semantics. In P.E. Dunne and TJ.M. Bench-Capon, editors, *Computational Models of Argument; Proceedings of COMMA 2006*, pages 121–130. IOS Press, 2006.

[5] M.W.A. Caminada. Comparing two unique extension semantics for formal argumentation: ideal and eager. In Mohammad Mehdi Dastani and Edwin de Jong, editors, *Proceedings of the 19th Belgian-Dutch Conference on Artificial Intelligence (BNAIC 2007)*, pages 81–87, 2007.

[6] M.W.A. Caminada and L. Amgoud. On the evaluation of argumentation formalisms. *Artificial Intelligence*, 171(5-6):286–310, 2007.

[7] M.W.A. Caminada, W.A. Carnielli, and P.E. Dunne. Semi-stable semantics. *Journal of Logic and Computation*, 22(5):1207–1254, 2012.

[8] M.W.A. Caminada, Samy Sá, and Joo Alcântara. On the equivalence between logic programming semantics and argumentation semantics. Technical Report ABDN–CS–13–01, University of Aberdeen, 2013.

[9] P. M. Dung, P. Mancarella, and F. Toni. Computing ideal sceptical argumentation. *Artificial Intelligence*, 171(10-15):642–674, 2007.

[10] P.M. Dung, R.A. Kowalski, and F. Toni. Assumption-based argumentation. In Guillermo Simari and Iyad Rahwan, editors, *Argumentation in Artificial Intelligence*, pages 199–218. Springer US, 2009.

[11] P.M. Dung. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artificial Intelligence*, 77:321–357, 1995.

[12] P.E. Dunne, W. Dvořák, and S. Woltran. Parametric properties of ideal semantics. *Artificial Intelligence*, 202:1–28, 2013.

[13] A.J. Garca, J. Dix, and G.R. Simari. Argument-based logic programming. In Guillermo Simari and Iyad Rahwan, editors, *Argumentation in Artificial Intelligence*, pages 153–171. Springer US, 2009.

[14] G. Governatori, M.J. Maher, G. Antoniou, and D. Billington. Argumentation semantics for defeasible logic. *Journal of Logic and Computation*, 14(5):675–702, 2004.

[15] S.J. Modgil and H. Prakken. A general account of argumentation with preferences. *Artificial Intellligence*, 2013. in press.

[16] H. Prakken. An abstract framework for argumentation with structured arguments. *Argument and Computation*, 1(2):93–124, 2010.

[17] G.R. Simari and R.P. Loui. A mathematical treatment of defeasible reasoning and its implementation. *Artificial Intelligence*, 53:125–157, 1992.

[18] F. Toni. Reasoning on the web with assumption-based argumentation. In Thomas Eiter and Thomas Krennwallner, editors, *Reasoning Web. Semantic Technologies for Advanced Query Answering*, pages 370–386, 2012.

[19] B. Verheij. Two approaches to dialectical argumentation: admissible sets and argumentation stages. In J.-J.Ch. Meyer and L.C. van der Gaag, editors, *Proceedings of the Eighth Dutch Conference on Artificial Intelligence (NAIC'96)*, pages 357–368, 1996.

[20] G.A.W. Vreeswijk. Abstract argumentation systems. *Artificial Intelligence*, 90:225–279, 1997.

[21] Y. Wu, M.W.A. Caminada, and D.M. Gabbay. Complete extensions in argumentation coincide with 3-valued stable models in logic programming. *Studia Logica*, 93(1-2):383–403, 2009. Special issue: new ideas in argumentation theory.